Wage Growth, Productivity Growth, and the Evolution of Employment

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Abstract

This paper studies the impact of wage growth on the evolution of employment in an intertemporal general-equilibrium model with endogenous productivity growth. For real wage growth above laissez-faire levels, we obtain steady-state equilibria in which productivity grows at the same rate as wages, the real interest rate is below the laissez-faire level, and so is the common growth rate of consumption, demand, and output. In these steady-state equilibria employment contracts at a constant rate equal to the difference between the growth rates of productivity and output. This contrasts with the view that equality of wage growth and productivity growth is a condition for constant employment.

Keywords: endogenous technical change, perfect competition, productivity growth, wages, employment

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1 Introduction

This paper takes a new look at the old question of what are the effects of statutory or union-imposed minimum wages on employment. We treat this as a macroeconomic question in the context of an intertemporal general-equilibrium model with endogenous productivity growth. We focus on the effects of wage growth on productivity growth and on the evolution of output and employment. Our results suggest that the terms of discourse about the matter change quite radically when it is studied in an intertemporal setting and technical change is seen as the result of optimizing behavior, responding to the incentives that are set by current and anticipated future market conditions.

The effects of wages on employment have traditionally been discussed in terms of cost effects versus aggregate-demand effects in a static, one-period setting with given technologies and given capital stocks. From a *Keynesian* perspective, involuntary unemployment is due to an insufficiency of aggregate demand for output, so if a wage increase induces workers and their families to raise their consumption, it may raise aggregate demand for output and therefore employment (see, e.g., Malinvaud (1977)). In contrast, from a *classical* perspective, wage increases merely raise marginal costs of production and reduce the incentive for employers to offer jobs.

The dichotomy of classical and Keynesian arguments plays a central role in political debate as well as applied research on the causes of involuntary unemployment. According to one view, the secular increase in European unemployment over the past three decades was due to expansive wage policies in the late sixties and early seventies opening a "gap" between actual wages and full-employment wages, which subsequently has never been closed (see, e.g., Bruno and Sachs (1985), Bentolila and Saint-Paul (1998)). According to another view, this unicausal explanation is at odds with the observation that in the first half of the eighties, unemployment in Europe was still dramatically going up even as wage policies had become less aggressive and labor shares were going down again; this observation is deemed to support a "Keynesian" explanation of the increase in European unemployment in the eighties as being due to a deficiency of aggregate demand (see, e.g., Drèze and Bean (1990), Blanchard (1997)). The increase in European unemployment in the eighties is also related to an insufficiency of capital, deemed to be the more serious as technologies used in the eighties seem to have become more capital intensive.[1]

[1] See, e.g., Blanchard (1997). As of the mid-eighties already, the German Council of Economic Experts (Sachverständigenrat) noted that idle-capacity rates reported by firms

As indicated by this account, the question of what are the effects of wage policies on unemployment is very much a "European question". The notion of a "wage policy" itself would seem to be meaningless in an economy in which labor markets are unregulated and competitive, and wage rates are market-determined. It is however very relevant for economies where this is not the case, i.e., economies with strong labor unions and/or significant labor market regulation in which wage rates are largely determined outside of competitive markets.

Even for such economies, one may wonder about the significance of a comparative-statics analysis of the effects of wage policies, i.e., an analysis which takes wage policies as given and considers their effects on, e.g., aggregate output and employment. After all, even if wages are determined outside competitive markets, "wage policies" will not really be exogenous as, e.g., the labor unions will take account of current labor market conditions. To the extent that they do so, the mere analysis of the effects of wage policies may be considered unsatisfactory as it says nothing about the determination of these policies and neglects the potential feedback effects from observed unemployment to the results of collective wage bargaining.

However the feedback effects from current market conditions to a labor union's bargaining stance will depend on assessments of how different wage policies will affect macroeconomic aggregates, in particular the level of unemployment. A "Keynesian" labor union may well consider that high unemployment calls for large wage increases because such wage increases raise aggregate demand and hence output. Even if one appreciates the ultimate endogeneity of wage policies, the simple comparative-statics question we pose, which takes wage policies as given, is of interest because the answers to this question will affect the different participants' preferences over wage policies.

At another level, the effects of wage policies on employment are a concern of political discussion. In the background of the actual wage setting, this political discussion is important because it influences assessments of legitimacy of the different parties' bargaining stances. These assessments of legitimacy in turn are important because at least some of the power of wage-setting institutions, i.e., unions and employers' associations, depends on legislation, statutory regulation, and jurisdiction; this support of the wage-setting institutions' power would be endangered if the wage policies that are implemented were widely perceived as outrageous.

In such political discussions, economists relying on the classical approach frequently refer to a benchmark wage policy which would have real wages

seemed to become dissociated from unemployment rates, so that, in contrast to previous decades, full capacity utilization was no longer a guarantee of full employment.

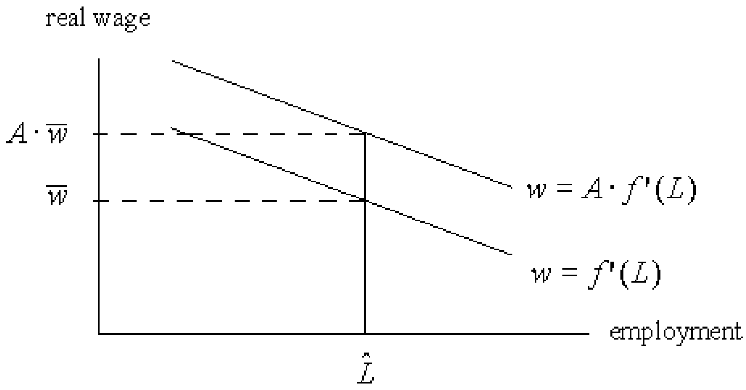

Figure 1: The classical "employment-neutral" benchmark.

grow at the same rate as the productivity of labor. Such a policy is deemed to leave employment unchanged, because, as illustrated in Figure 1, multiplication of the marginal product of labor and the real wage rate by the same factor $A > 1$ has no effect on the demand for labor by profit-maximizing firms.[2] In Germany for instance, the Council of Economic Experts (Sachverständigenrat) regularly uses this wage policy as a benchmark, with the addendum that unemployment will be reduced if and only if the growth of real wages falls short of this benchmark. The political appeal of such recommendations is somewhat weakened if, as happened in the first half of the eighties and again in recent years, a few years of real wage growth below productivity growth are accompanied by significant *increases* in unemployment (see again Blanchard (1997) or Bentolila and Saint-Paul (1998)). The question of what are the effects of wage policies on employment and what is a suitable benchmark for assessing wage policies is therefore very much up in the air.

The approach we use to address these questions has three distinct features:

- We look at an intertemporal model in which current choices in any period are affected by anticipated future wage rates as well as current wage rates.
- We treat productivity growth as the result of innovation investments.

[2] For a textbook treatment, see, e.g., Branson (1989), pp. 480 ff.

- We give a full account of all general-equilibrium repercussions, including repercussions arising from income and wealth effects.

We define wage policies in terms of real wage rates. Whereas Keynes (1936) insisted that labor contracts are concluded in terms of money wages, in fact, the parties concluding these contracts evaluate them in terms of their anticipated implications for real wages. Discussions of benchmark policies involving real wage growth equal to productivity growth or money wage growth equal to inflation plus productivity growth suggest that there is not much money illusion in wage determination. Assuming that expectations about the goods prices that are implied by different wage rates are rational, we consider real-wage policies to be the proper object of analysis. The model we use does not even involve money so that our analysis is unencumbered by the various aspects of money illusion, mistaken inflationary expectations, etc. which have dominated the literature on wage policy versus monetary policy from Keynes (1936) and Modigliani (1944) to the sixties' and seventies' debates about expectations formation and the Phillips Curve (see, e.g., Friedman (1968), Lucas (1973)).

In our setting we find that the dichotomy of classical and Keynesian, cost and demand, arguments is misplaced. *Methodologically*, we confirm the Keynesian view that the effects of wages on employment depend on the behavior of aggregate demand and cannot be determined merely by looking at the profit-maximizing choices of firms. This is true even though wage policies are defined in terms of *real* wages and, in contrast to Keynes's own discussion in Chapter 19 of the *General Theory*, there is no need for aggregate-demand considerations to determine the goods prices and hence the real wage that go with a given money wage. However, in *substantive terms*, our analysis leads to an assessment of the effects of wages on employment that is even more pessimistic than the traditional classical view. Overexpansionary wage policies induce productivity growth that outpaces the growth of aggregate demand; employment will then be continually shrinking even though real wages and productivity grow at the same rate, and the wage policy is conforming to the condition for employment neutrality mentioned above.

The need to allow for aggregate-demand repercussions of wage policies arises from the following features of our model:

- Current labor is not the only input into current production. With labor productivity in any given period determined by past innovation investments, these innovation investments must be seen as additional inputs.

- Prices of inputs other than labor are free to adjust without frictions. This is true, in particular, of real interest rates, which determine the cost of innovation investments in terms of the subsequent period's output.

- When all inputs are taken into account, the technology exhibits constant returns to scale at the level of economic aggregates.

As in the standard model of general equilibrium with linear production (see, e.g., Arrow and Hahn (1972), Bliss (1975)), these features of our model imply that the vector of all input prices together adjusts so that firms make zero profits; the most immediate effects of current and anticipated future statutory or union-set minimum real wages will therefore be on the relative prices of inputs other than labor, here on real interest rates. Given that this adjustment of other input prices occurs and maximum profits of firms are zero, aggregate production - and with it aggregate employment - are demand-determined: With constant returns to scale, the production sector of the economy does not care at what scale it produces and earns zero profits.

The usual textbook analysis of the effects of wages on employment tends to neglect the role of inputs other than labor, taking, e.g., the firm's endowments of know-how and capital as given. For a static analysis of effects arising in a single period this is unproblematic. In an intertemporal setting, one has to take account of the fact that investment choices and intertemporal prices in any given period will depend on people's anticipations of wages and prices in subsequent periods. To the extent that minimum-real-wage policies affect these anticipations as well as the wage rates that are actually paid, a complete analysis has to take account of the overall effects of wage policies on the time paths of equilibrium intertemporal prices, investment choices, and incomes. With constant returns to scale on aggregate, a standard assumption of intertemporal macroeconomic models, one is led to the twin conclusion that *(i) the time paths of wages, prices, and interest rates together must satisfy a zero-profit condition, and (ii) the time paths of output and employment must be demand-determined.*

However, in an intertemporal setting, the aggregate demand for manufactured goods in any one period will not be "autonomous" in the Keynesian sense, but will itself be the result of intertemporal optimization and coordination of households and firms. A purchasing-power effect of high real wages may be present, but then the question is how this effect is spread over time. This depends on intertemporal prices and on the market participants' responses to the incentives set by these prices.

In our analysis the most dramatic effects of wage policies on the evolution of output and employment arise because *the wage-induced adjustments in* *equilibrium interest rates distort the allocation of aggregate goods demand over time.* If real interest rates have to adjust so that firms earn zero profits even at high real wages this may make it impossible for interest rates to bring the consumption growth that is desired by households into line with the output growth which can be provided by workers and firms. The growth of aggregate demand may then fall short of productivity growth, and there may be a *continuous decline of employment* as the economy seems to be running out of jobs. As wage policies affect interest rates, the economy's intertemporal coordination mechanisms and the intertemporal allocation of resources suffers.

We study these interdependencies in terms of the *steady-state equilibria* of a model in which current labor is the only input into current production, but prior investments serve to improve the productivity of labor from one period to the next. The model is a simplified version of the model of "Endogenous Technical Change in a Competitive Economy" that we studied in Hellwig and Irmen (1999), see also Bester and Petrakis (1998). Taking the *growth rate* of minimum real wages as the key policy variable, we ask how this variable affects the evolution of actual wages, productivity, aggregate consumption, output, and employment. In Hellwig and Irmen (1999) we have shown that under *laissez-faire*, the intertemporal price system would ensure that the growth of aggregate demand is compatible with wage growth and productivity growth so that, e.g., with a constant labor force one has full employment all the time. In contrast this paper shows that if the policy-imposed growth rate of minimum real wage rates exceeds the rate that would prevail under *laissez-faire*, the steady-state equilibrium productivity growth rate will be the same as the wage growth rate, but the growth rate of aggregate demand will be less, and *employment will be contracting at a constant rate that is roughly equal to the difference between the growth rate of real wages and the growth rate of aggregate demand.* Preliminary research that we have done on non-steady-state equilibria suggest that these conclusions are not limited to steady-states, but can be extended to long-run averages of growth rates of non-steady-state equilibria.

At this point one may want to go back to the specification of wage policies. As mentioned above, we do not provide any descriptive account of how wage policies are chosen. But if employment is contracting at a constant rate, shouldn't we expect the wage-setting mechanism to be changed? Shouldn't we expect that either the wage-setting institutions will choose a different wage policy or the political system will intervene to dismantle the power of the wage-setting institutions?

The answer to these questions depends on whether in a situation like the one we describe the continually worsening unemployment problem is linked to wage policy. This in turn depends on the theoretical framework that underlies people's thinking about wages and employment. On the basis of a static framework with exogenous productivity growth, even an economist who takes the classical approach will note that real wages and productivity are growing at the same rate and hence that wage policy is probably *not* responsible for the ongoing decline in employment. If so, why should one bother about wage policies? Why not look elsewhere and, e.g., look for policy measures that will correct the insufficiency of growth in aggregate demand?[3] With the traditional separation of demand analysis and supply analysis, one may not in fact appreciate that this insufficiency itself may be due to excessive wage growth and to the distortions in intertemporal pricing that this induces.

The evolution of European unemployment over the past three decades provides a case in point. From the perspective of our analysis, the development of the early eighties that Drèze and Bean (1990) or Blanchard (1997) interpret in Keynesian terms may in fact be a delayed response to the high wage growth (and low real interest rates) of the seventies. Declining labor shares in the early eighties may reflect the effects of the previous wage push on the development of labor productivity; the deficiency of aggregate demand as well as the deficiency of capital in a situation with more capital-intensive techniques of production may reflect the very same effects.[4] With this interpretation, the distinction between the "classical increases in unemployment" in the seventies and the "Keynesian increases in unemployment in the eighties" would become moot.

Whether this is *in fact* a satisfactory account for the evolution that took place is an empirical question that transcends the scope of our paper. Our contribution is merely to show that this question needs to be asked. More generally, the contribution of our paper is to redefine the conceptual framework within which the effects of wage policies on the overall evolution of the economy are discussed.

Our basic model is sketched in Section 2. The core of the analysis is presented in Section 3. We restrict attention to the analysis of steady-state equilibria. In Section 4 we discuss further aspects of our analysis: implica-

[3]For an overview of policy measures considered to alleviate European unemployment, see Drèze and Malinvaud (1994).

[4]The possibility of a link between the wage push of the seventies and the change of technology in the early eighties has previously been suggested by Caballero and Hammour (1997). Staying within a partial-equilibrium setting, they discuss technology choice in terms of its prospective effects on the relative power of capital and labour in subsequent wage bargaining rather than the more immediate enhancement of labour productivity studied in this paper.

tions for economic policy and empirical research, the scope for extending the results to non-steady-state equilibria, and finally the scope for endogenizing wage policies.

2 The Model

As in Hellwig and Irmen (1999), we study an economy with a household sector and a production sector, with three objects of exchange, a manufactured good, labor, and bonds, in an infinite sequence of periods $t = 1, 2, ...$ The manufactured good serves for investment as well as consumption. In each period t, there are markets for the three objects of exchange. Treating the manufactured good as the numéraire, we let w_t denote the real wage and p_t^b the real bond price at t. A bond at t is defined as a claim on one unit of the manufactured good at $t+1$. Working with real interest rates rather than real bond prices, we write $p_t^b = 1/(1 + r_t)$ where r_t is the real interest rate from period t to period $t + 1$.

2.1 The Household Sector

For simplicity we assume that the household sector comprises one household.[5] This household has an initial endowment of B_0 bonds coming due at $t = 1$, L units of labor in each period $t = 1, 2, ...,$ and 100% of the shares of all firms. The household draws utility from his consumption c_t in periods $t = 1, 2, ...,$ evaluating the sequence $\{c_t\}$ according to the functional

$$\sum_{t=1}^{\infty} \beta^t \ln c_t, \tag{1}$$

where $0 < \beta < 1$ is a discount factor. The household does not care about leisure.

The household chooses a strategy for his consumption demand c_t, labor supply L_t, and bond demand B_t^d in all periods $t = 1, 2, ...$ In choosing his strategy, he takes account of his given initial endowment as well as the levels of real wages w_t and real interest rates r_t that he expects to prevail in periods $t = 1, 2, ...$ He also forms expectations about potential quantity constraints $\widehat{L}_t$ on employment and about aggregate real dividend distributions Π_t in

[5] The reader may be uncomfortable with the assumption of one infinitely-lived household. However, it is easy to verify that our results apply just as well to an overlapping-generations model with finite lifetimes of households.

periods $t = 1, 2, ...$, and takes account of these expectations as he chooses his strategy. Taking all these data as given he chooses his strategy so as to maximize the utility functional (1) under the constraints that

$$c_t + B_t^d/(1 + r_t) = w_t L_t + B_{t-1}^d + \Pi_t, \tag{2}$$

$$c_t \geq 0,\ L_t \leq \min\left(\widehat{L}_t, L\right),\ B_t^d \geq 0 \tag{3}$$

for all t, with $B_0^d = B_0$, given.

We do not need to go into the details of this maximization. The following observations contain all we need for our analysis:

- Given that the household does not care about leisure, he always desires to supply as much labor as possible, i.e., he sets

$$L_t = \min\left(\widehat{L}_t, L\right) \tag{4}$$

  for all t. The quantity constraint on employment is binding whenever $\widehat{L}_t$ is less than L; the difference $L - \widehat{L}_t$ provides a measure of involuntary unemployment of the household.

- The household's budget set is unbounded and his maximization problem fails to have a solution if the series giving the discounted present value of his wage incomes in all periods,

$$w_1 L_1 + \sum_{t=2}^{\infty} \prod_{i=1}^{t-1} \left(\frac{1}{1 + r_i}\right) w_t L_t, \tag{5}$$

  fails to converge.

- The first–order conditions for the household's choice of c_t, B_t^d, and c_{t+1} yield the usual Euler equation

$$c_{t+1} \geq \beta(1 + r_t)\, c_t, \tag{6}$$

  with a strict inequality only if $B_t^d = 0$,

  for the growth of the household's consumption, linking desired consumption growth to the discount factor and the real interest rate. This condition will be seen to play a crucial role for aggregate-demand and employment dynamics.

2.2 Firms, Technologies, and Profit Maximization

The production sector of the economy is represented by an atomless measure space of firms. With respect to the production of output in periods $t = 2, 3, ...$, all firms have the same technology. Each firm has a capacity limit of one unit of output per period.[6] Its output in period t is given as

$$y_t = \min(1, a_t l_t), \tag{7}$$

where l_t is the firm's labor input and a_t its labor productivity in period t. The firm's labor productivity a_t is equal to

$$a_t = A_{t-1}(1 + q_t); \tag{8}$$

here A_{t-1} is an indicator of economy-wide labor productivity in period $t-1$, and q_t is an indicator of productivity growth at this firm.

To achieve the productivity growth rate q_t from period $t-1$ to period t, the firm must invest $K(q_t)$ units of the manufactured good in period $t-1$.[7] The resulting innovation is assumed to be proprietary knowledge of the firm in period t, i.e., the period when it is made. Subsequently, as we discuss below, the innovation becomes embodied in the economy-wide productivity indicators $A_t, A_{t+1}, ...$, with no further scope for proprietary exploitation. The function $K(.)$ is assumed to be strictly increasing, convex, and continuously differentiable; moreover, $K(0) = 0$.

The innovation investment $K(q_t)$ in period $t-1$ is financed by an issue of $(1 + r_{t-1})\ K(q_t)$ bonds. In terms of the manufactured good of period t as numéraire, a production plan (q_t, l_t, y_t) for period t thus yields the profit

$$\begin{aligned} \pi_t &= y_t - w_t l_t - (1 + r_{t-1})K(q_t) \\ &= \min[1,\ A_{t-1}(1 + q_t)l_t] - w_t l_t - (1 + r_{t-1})K(q_t) \end{aligned} \tag{9}$$

where $y_t = \min[1,\ A_{t-1}(1+q_t)l_t]$ is the firm's revenue from output sales, $w_t l_t$ its labor cost at the real wage rate w_t, and $(1 + r_{t-1})K(q_t)$ its debt service. The profit, if any, is immediately distributed to the household as the firm's shareholder.

[6] In Hellwig and Irmen (1999) we consider a more general specification involving variable capacity based on prior capacity investments, with investment outlays a strictly convex function of capacity. The analysis here is easily extended to this more general model. The simpler specification treated here requires significantly less notation.

[7] Alternatively we might assume that inputs into innovative activity take the form of labor. In this case the main conclusions of our analysis would still be valid if capacity constraints were defined in terms of unit of labor, e.g., each firm can employ up to one unit of labor.

We assume that the firm takes the sequence $\{w_t, r_t\}$ of real wages and interest rates as well as the sequence $\{A_t\}$ of aggregate productivity indicators as given and chooses its production plan so as to maximize the sum of the discounted present values of its profits in all periods. Because production choices for different periods are independent of each other, for each period t, it will in fact choose the plan (q_t, l_t, y_t) to maximize the profit π_t from this plan in period t.

Notice that the firm's technology involves a nonconvexity: The cost $(1 + r_{t-1})K(q_t)$ that is associated with a given innovation rate $q_t > 0$ is fixed, i.e., independent of the output y_t that the firm produces. This introduces a positive scale effect, namely if the firm innovates at all, then it wants to apply the innovation to as large an output as possible and to produce at the capacity limit $y_t = 1$. The chosen input combination (q_t, l_t) must then be minimizing the costs of producing the capacity output. However, depending on the price variables w_t, r_{t-1}, and the productivity index A_{t-1}, it may be the case that minimum unit costs of production exceed one, i.e., the price of output as the numéraire; in this case the firm will prefer not to produce any output at all.

Minimization of unit costs of production by the input combination (q_t, l_t) requires

$$w_t l_t = \frac{w_t}{A_{t-1}(1+q_t)}, \tag{10}$$

and

$$q_t \in \arg\min_{q \geq 0} \left[\frac{w_t}{A_{t-1}(1+q)} + (1 + r_{t-1})K(q) \right]. \tag{11}$$

Given the differentiability and convexity of the innovation cost function $K(.)$, (11) actually determines q_t *uniquely* as

$$q_t = q^* \left(\frac{w_t}{A_{t-1}(1 + r_{t-1})} \right), \tag{12}$$

where, for any w_t, r_{t-1}, and A_{t-1}, $q^*(w_t/A_{t-1}(1 + r_{t-1}))$ is defined as the solution to the first-order condition

$$\frac{w_t}{A_{t-1}(1+q^*)^2} \leq (1 + r_{t-1})K'(q^*),$$
$$\text{with strict inequality } \textit{only} \text{ if } q^* = 0, \tag{13}$$

which is obviously unique.

Let

$$K^*\left(\frac{w_t}{A_{t-1}}, r_{t-1}\right) := \min_{q \geq 0}\left[\frac{w_t}{A_{t-1}(1+q)} + (1+r_{t-1})K(q)\right] \tag{14}$$

be the minimal unit cost of production of the firm in period t. Depending on how for given w_t, r_{t-1}, and A_{t-1}, $K^*(w_t/A_{t-1}, r_{t-1})$ relates to the price of output, profit-maximizing production plans for period t may take three distinct forms:

- A production plan (q_t, l_t, y_t) with capacity production, $y_t = 1$, maximizes a firm's profits if and only if the input choice (q_t, l_t) satisfies (10) and (12), and moreover $K^*(w_t/A_{t-1}, r_{t-1}) \leq 1$.
- A production plan (q_t, l_t, y_t) with positive production below capacity, $y_t \in (0, 1)$, maximizes a firm's profits if and only if the input choice (q_t, l_t) satisfies $q_t = q^*(w_t/A_{t-1}(1 + r_{t-1})) = 0$, $l_t = y_t/A_{t-1}$, and moreover $K^*(w_t/A_{t-1}, r_{t-1}) = 1$.
- A production plan (q_t, l_t, y_t) with zero production, $y_t = 0$, maximizes a firm's profits if and only if $q_t = 0$, $l_t = 0$, and moreover, $K^*(w_t/A_{t-1}, r_{t-1}) \geq 1$.

For any constellation of the parameters w_t, r_{t-1}, and A_{t-1}, there may be more than one profit-maximizing production plan. In particular, if we have $K^*(w_t/A_{t-1},\ r_{t-1}) = 1$, maximum profits are zero, and this maximum is attained at both, the plan (q_t, l_t, y_t) satisfying (10), (12), and $y_t = 1$, *and* the plan $(0, 0, 0)$ providing for inactivity of the firm in period t. If in addition $q^*(w_t/A_{t-1}(1 + r_{t-1})) = 0$, profits are maximized by *any* production plan of the form $(0, y/A_{t-1}, y)$.

However all profit-maximizing plans with positive production in period t will involve the same innovation rate q_t. If $K^*(w_t/A_{t-1}, r_{t-1}) \leq 1$ and $q^*(w_t/A_{t-1}(1 + r_{t-1})) > 0$, there actually is only one profit-maximizing plan with positive production. If $K^*(w_t/A_{t-1}, r_{t-1}) = 1$ and $q^*(w_t/A_{t-1}(1 + r_{t-1})) = 0$, there are multiple profit-maximizing plans with positive production, but all of them involve the same innovation rate $q^*(w_t/A_{t-1}(1+r_{t-1})) = 0$. Given that all firms have the same technology, this observation implies that all active firms in a given period t will have the same innovation rate q_t.

2.3 The Production Sector as a Whole

The set of all firms is represented by the set $\Re_+$ of nonnegative real numbers. The weight of any one subset of firms relative to the household (sector) is given by its Lebesgue measure. For instance, if all firms that are active in period t have the same production plan (q_t, l_t, y_t) for this period, and if the set of these firms has Lebesgue measure n_t, we say that the aggregate investment demand of firms in period $t-1$ is $n_t K(q_t)$, aggregate labor demand in period t is $n_t l_t$, and aggregate goods supply in period t is $n_t y_t$. To assess the market impact of these production choices, these aggregate quantities must be compared to the household's consumption demand and labor supply.

We assume that if firms choose to be active, they always plan to produce the capacity output $y_t = 1$. This assumption simplifies the exposition because it implies that active firms all choose the same production plan (q_t, l_t, y_t) for period t; accordingly their market impact can be represented in the form $n_t K(q_t)$, $n_t l_t$, $n_t y_t$, where n_t is the measure of the set of active firms and indeed $n_t y_t = n_t$. No significant loss of generality is involved because in those circumstances where active firms do plan to produce some output $y_t \neq 1$, their maximized profits as well as their innovation investments are zero, and they would be just as willing to choose the production plan $(0, 1/A_{t-1}, 1)$ or the production plan $(0, 0, 0)$. It would therefore be possible to rearrange profit-maximizing production plans across firms so that all firms plan to have output equal to either zero or one and moreover the aggregate impact of firms on markets is unchanged.

In representing the set of all firms by $\Re_+$ with Lebesgue measure, we implicitly introduce a zero-profit condition. Given that labor supply in each period is bounded, in any equilibrium the set of firms employing more than some $\varepsilon > 0$ units of labor must have bounded measure and hence must be smaller than the set of all firms. Given that the inactive firms must be maximizing profits just like the active ones, this implies that in any equilibrium in any period t, $t = 2, 3, ...$, maximum profits of firms at equilibrium prices must be equal to zero.

So far we have not said anything about production in period 1. Taking our cue from the preceding account of firm behavior in periods $2, 3, ...$, we assume that as of period 1, there is a given set of measure $n_1 > 0$ of firms that have all made the same prior innovation investment $K(q_1)$ and now have the same labor productivity $a_1 = A_0(1 + q_1) > 0$; these firms also have outstanding debt obligations equal to B_0 on aggregate, or B_0/n_1 per firm; this is the counterpart of the household's initial holdings of bonds.

At this point we should in principle allow for firms in period 1 that have not made any prior innovation investments to be able to supply the manufactured good in period 1 with the old technology, with labor productivity A_0. This would require us to distinguish two distinct sets of active firms in period 1. However we neglect this possibility and simply assume that apart from the given set of firms with measure n_1 no additional firm wants to supply output in period 1.[8]

To conclude the account of the production sector, we turn to the evolution of the economy-wide productivity indicators $A_0, A_1, \ldots$ Given that for any t all firms that are active at t choose the same innovation rate q_t and attain the same labor productivity $a_t = A_{t-1}(1+q_t)$, we identify A_t with a_t and write

$$A_t = A_{t-1}(1+q_t) \tag{15}$$

for $t = 1, 2, \ldots$, with $A_0 > 0$ and q_1 given by initial conditions. This specification reflects the assumption mentioned above that all innovations are publicly available after one period. Anybody can then incorporate them into their production processes or take them as a basis for additional innovations. Proprietary use of innovations is thus limited to the period in which they occur. During this period, the benefits they provide are somewhat diluted by the reduplication of innovative effort that takes place if many firms innovate at the same time, but, as discussed by Hellwig and Irmen (1999), with limited capacities of innovating firms, this reduplication does not eliminate the quasi-rents available to the innovators.

In specifying the dynamics of economy-wide productivity indices through (15), we are using (abusing of) the fact that under our assumptions, in any period t will involve all firms that are active in t will choose the same innovation rate q_t. In a more general setting, with genuinely heterogenous firms, we should have to provide a detailed account of the relation between the cross-section distribution of innovation choices of active firms and the induced spillovers into economy-wide productivity advances. There are several ways to do this, each one with its own advantages and disadvantages; for the purposes of this paper though, there is no need to dwell on these.

2.4 Intertemporal General Equilibrium

Turning to the behavior of the economy as a whole, we refer to a sequence $\{w_t, r_t\}$ of real wages and real interest rates for periods $t = 1, 2, \ldots$ as a *price system.* By an *allocation* we understand a sequence $\{c_t, L_t, B_t^d, n_t, q_t, l_t\}$ that comprises a strategy $\{c_t, L_t, B_t^d\}$ for the household and, for each t, a measure n_t of firms active at t, producing the capacity output $y_t = 1$ with input choices (q_t, l_t). Whereas in Hellwig and Irmen (1999), we had assumed that

[8] This requires $w_1 \geq A_0$, and will be true if A_0 is sufficiently small and n_1 and q_1 are sufficiently large. Alternatively, the presumption is that prior choices of firms planning for production in period 1 have been governed by the same principles as the choices of firms planning for production in later periods.

all price variables are fully flexible and all coordination of activities occurs through the price system, here we allow for downward rigidity of real wages providing a role for quantity constraints in the labor market.

For this purpose we introduce the notion of a *wage policy* as a sequence $\{\bar{w}_t\}$ of lower bounds on real wage rates in the market, and we assume that labor market institutions prevent market participants from violating these lower bounds. Given a wage policy $\{\bar{w}_t\}$, a *Keynes-Radner equilibrium* will correspond to a price system $\{w_t, r_t\}$, an allocation $\{c_t, L_t, B_t^d, n_t, q_t, l_t\}$, and a sequence $\{\widehat{L}_t, \Pi_t, A_t\}$ of quantity constraints on the household's employment, distributed aggregate profits, and productivity indicators that satisfy the following conditions:

(E1) Given the initial bond endowment B_0 and the sequence $\{w_t, r_t, \widehat{L}_t, \Pi_t\}$ of real wages, interest rates, quantity constraints on employment $\widehat{L}_t$, and dividend distributions Π_t, the strategy $\{c_t, L_t, B_t^d\}$ for the household maximizes his utility (1) under the constraints (2) and (3), with the initial condition $B_0^d = B_0$.

(E2) For any t, the profit distribution Π_t which the household expects to receive at t is equal to the actual aggregate of profits of firms active at t, i.e.,

$$\Pi_t = n_t[1 - w_t l_t - (1 + r_{t-1})K(q_t)]. \tag{16}$$

(E3) Given the productivity indicator A_{t-1}, the real wage rate w_t, and the real interest rate r_{t-1}, for any $t > 1$, the input choice $(q_t, l_t, 1)$ minimizes the unit cost of production of a firm active at t.

(E4) Given the productivity indicator A_{t-1}, the real wage rate w_t, and the real interest rate r_{t-1}, for any $t > 1$,

$$K^* \left(\frac{w_t}{A_{t-1}}, r_{t-1} \right) \geq 1, \tag{17}$$

with a strict inequality only if $n_t = 0$.

For $t = 1$, $w_1 \leq A_0(1 + q_1)$.

(E5) For any t,

$$c_t + n_{t+1} K\left(q_{t+1}\right) = n_t. \tag{18}$$

(K6) For any t,

$$w_t \geq \bar{w}_t, \tag{19}$$

$$\widehat{L}_t = n_t l_t \leq L, \tag{20}$$

and

$$(w_t - \bar{w}_t)(L - \hat{L}_t) = 0. \tag{21}$$

(E7) For any t,

$$B_t^d = (1 + r_t)\, n_{t+1} K(q_{t+1}). \tag{22}$$

(E8) For any t, the indicators A_t satisfy the updating condition (15).

Except for the labor market condition (K6), these are the usual conditions for Radner's (1972) "equilibrium of plans, prices and price expectations", adapted to the present model; a detailed discussion is given in Hellwig and Irmen (1999). (E1), (E3), and (E4) ensure that the equilibrium allocation is consistent with utility maximization of the household and profit maximization of firms, those that are active and those that are not. (E2) ensures that the household's expectations about profits and dividends are consistent with firms' production plans, (E5) and (E7) impose market clearing for manufactured goods and bonds, and (E8) reasserts the link between innovative activity and economy-wide productivity growth.

The labor market condition (K6) should be contrasted with the full-employment condition

(E6) For any t,

$$\widehat{L}_t = n_t l_t = L. \tag{23}$$

Condition (E6) provides for market-clearing in the labor market. This would be appropriate if wages were fully flexible. In contrast, condition (K6) takes account of the restrictions that are imposed by the given wage policy. If the minimum-wage condition (19) is binding, employment may fall short of notional labor supply L, and there may be involuntary unemployment. This is why the equality in (23) is replaced by an inequality in (20); as indicated by (21), the inequality in (20) can only be strict if $w_t = \bar{w}_t$.

We are interested in the effects of wage policies on Keynes-Radner equilibria, in particular, on the evolution of productivity, output, and employment. For simplicity we consider wage policies $\{\bar{w}_t\}$ that take the form

$$\bar{w}_t = \bar{w}_1 \, (1 + g_{\bar{w}})^{t-1} \tag{24}$$

for some initial wage level $\bar{w}_1 > 0$ and some growth rate $g_{\bar{w}} > 0$. We are particularly interested in the effect of the growth rate $g_{\bar{w}}$ of the minimum real wage on the intertemporal structure of the equilibrium allocation.

3 Steady-State Equilibria

To focus on economic essentials, we look at steady-state equilibria. A Keynes-Radner equilibrium will be referred to as a *steady-state equilibrium* if the equilibrium allocation $\{c_t, L_t, B_t^d, n_t, q_t, l_t\}$ has the property that productivity grows at a constant rate over time. As will be seen shortly, this automatically implies that the real interest rate as well as the growth rates of consumption, aggregate output, employment, and real bond issues are also constant over time.

In looking at steady-state equilibria we presume that the initial data B_0, $A_1 = A_0(1 + q_1), n_1$, and $\bar{w}_1$ take suitable values. Such an analysis is not fully general. This disadvantage though is compensated by the gain in transparency of economic interdependencies that comes from looking at steady-state equilibria rather than arbitrary non-steady-state equilibria. The scope for extending the analysis to non-steady-state equilibria will be considered in section 4.3 below.

The analysis of steady-state equilibria hinges on three conditions, the first-order condition (6) for the household's optimal choice of consumption in successive periods, the first-order condition (13) for the firms' optimal choice of an innovation rate, and the free-entry condition (E4). Let $\hat{q}$ be the steady-state growth rate of productivity. If $\hat{q}$ is positive,[9] the firms' first-order condition (13) must hold with equality and (E4) requires that $K^*(w_t/A_{t-1}, r_{t-1}) = 1$. These two conditions can be combined to yield

$$\frac{w_t}{A_{t-1}} = \frac{(1+\hat{q})^2 K'(\hat{q})}{h(\hat{q})} \tag{25}$$

and

$$r_{t-1} = \frac{1}{h(\hat{q})} - 1, \tag{26}$$

where $h(\hat{q}) := (1+\hat{q})K'(\hat{q}) + K(\hat{q})$. Upon combining (26) with the household's first-order condition (6) and noting that (E7) implies $B_t^d > 0$, one further

[9] Given that $g_{\bar{w}} > 0$, a steady-state innovation rate $\hat{q} = 0$ would imply that from some period on the unit costs of production exceed one, so profits are maximized at the production plan $(0, 0, 0)$ and the entire economy would be inactive.

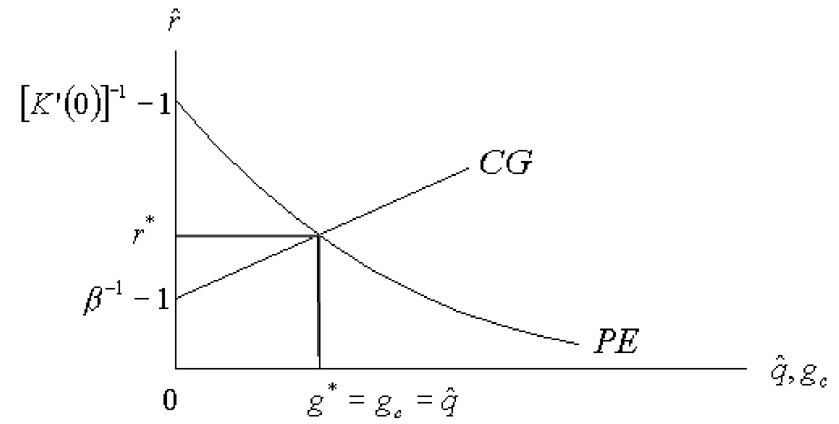

Figure 2: Steady-state equilibrium with $g_{\bar{w}} \leq g^*$.

obtains

$$\frac{c_{t+1}}{c_t} = \frac{\beta}{h(\hat{q})}. \tag{27}$$

Thus, in a steady-state equilibrium, the ratio w_t/A_{t-1}, the real interest rate, and the growth rate $g_c = \frac{c_{t+1}}{c_t} - 1$ of equilibrium consumption must all be constant over time.

A key question is how the growth rate g_c of equilibrium consumption relates to the rate of productivity growth $\hat{q}$. Clearly, since $0 \leq c_t \leq n_t \leq A_t L$ for all t, g_c cannot exceed $\hat{q}$. However g_c may be less than $\hat{q}$. Indeed we claim that g_c *must* be less than $\hat{q}$ if the growth rate $g_{\bar{w}}$ of the minimum real wage rate in (24) is sufficiently high. To see this, note that in a steady-state equilibrium the real wage rate w_t must grow at the same rate as labor productivity; this follows from the constancy of w_t/A_{t-1} that is given by (25). In view of (19) and (24), this in turn implies that $\hat{q} \geq g_{\bar{w}}$. If $g_{\bar{w}}$ is very large, it follows that $\hat{q}$ must also be very large, and, by (27), the consumption growth rate g_c must be very small, perhaps even negative. Hence if $g_{\bar{w}}$ is sufficiently large, g_c must be less than $\hat{q}$.

The interdependence between the steady-state equilibrium real rate of interest $\hat{r}$ and the growth rates of labor productivity and consumption is illustrated in Figures 2 and 3. The *decreasing* curve PE with vertical intercept $[K'(0)]^{-1} - 1$ corresponds to the production equilibrium condition (26), with $r_{t-1} = \hat{r}$, for the real rate of interest that is compatible with profit maximization and zero profits of firms at the innovation rate $\hat{q}$. The *increasing* curve

CG with intercept $\frac{1}{\beta} - 1$ corresponds to the equation

$$\hat{r} = \frac{1 + g_c}{\beta} - 1 \tag{28}$$

for the real interest rate that is compatible with the household's optimal consumption growing at the rate g_c. The figure is drawn under the assumption that $K'(0) < \beta$. Under this assumption, one easily sees that the two curves have a unique intersection point (g^*, r^*) with $g^* > 0$ and $\frac{1}{\beta} - 1 < r^* < [K'(0)]^{-1} - 1$.[10] Analytically, the intersection point is given by the equation

$$1 + g^* = \frac{\beta}{h(g^*)}, \tag{29}$$

which results from (26) and (28) by setting $\hat{q} = g_c = g^*$ and eliminating the interest rate $r_{t-1} = \hat{r}$. Given that $K(.)$ is increasing and convex, the right-hand side of (29) is decreasing in g^*, and there exists no more than one value of g^* for which this equation can hold.

In Theorem 1 below we show that the intersection point (g^*, r^*) in Figure 2 corresponds to a steady-state equilibrium if and only if the growth rate $g_{\bar{w}}$ of minimum real wages is no greater than g^*. If $g_{\bar{w}} > g^*$, the pair (g^*, r^*) *cannot* correspond to a steady-state equilibrium. In this case a steady-state Keynes-Radner equilibrium must involve the productivity growth rate $\hat{q} = g_{\bar{w}} > g^*$. Compatibility with profit maximization as well as free entry and exit of firms then requires that the real interest rate be sufficiently low, namely that $\hat{r} = [h(g_{\bar{w}})]^{-1} - 1$, as indicated by (26). This is less than r^*, so (28) shows that the induced growth rate of equilibrium consumption will be less than g^* and *a fortiori* less than $\hat{q} = g_{\bar{w}}$. Figure 3 provides a summary illustration of these considerations. As the figure is drawn, the equilibrium consumption rate is positive. However there is nothing to prevent it from being negative; indeed g_c *will* be negative if $g_{\bar{w}} = \hat{q}$ is sufficiently large and, correspondingly, the real interest rate $\hat{r}$ is sufficiently small.

What happens with aggregate output and employment when the growth rate of equilibrium consumption is less than the growth rate of labor productivity? We claim that *the steady-state equilibrium growth rates of aggregate*

[10] For real interest rates above $[K'(0)]^{-1} - 1$, profit maximization *cum* free entry and exit of firms entail $\hat{q} = 0$, i.e., the curve corresponding to (26) continues on the vertical axis. Therefore if $K'(0) \geq \beta$, the two curves in Figure 1 would intersect at $g^* = 0, r^* = \frac{1}{\beta} - 1$, which automatically implies $g^* < g_{\bar{w}}$.

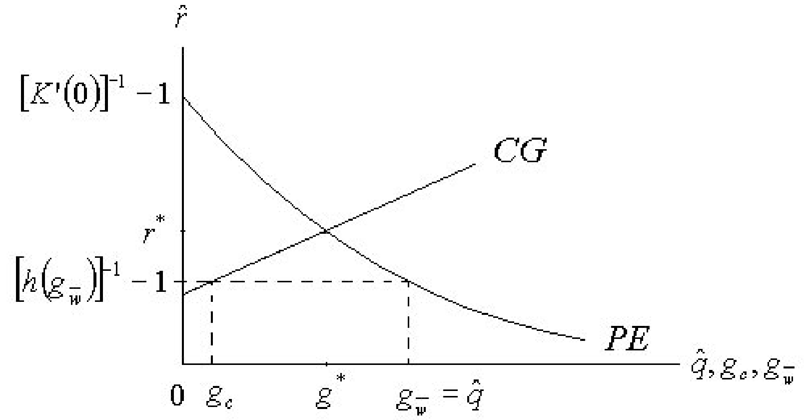

Figure 3: Steady-state equilibrium with $g_{\bar{w}} = \widehat{q} > g_c$.

output and wage income must be the same, and must coincide with the growth rate g_c of consumption. Equality of the growth rates of aggregate output and wage income follows from the observation that, by successive use of the household's labor supply condition (4), the labor market condition (20), the cost minimization condition (10), and (25), with $q_t = \hat{q}$, one has

$$w_t L_t = w_t \hat{L}_t = w_t \; l_t \; n_t = \frac{w_t}{A_{t-1}(1+q_t)} \; n_t = \delta(\hat{q}) \; n_t, \tag{30}$$

where

$$\delta(\hat{q}) := \frac{(1+\hat{q})K'(\hat{q})}{h\,(\hat{q})} \tag{31}$$

is the steady-state equilibrium share of wages in output.

As for the relation between the common growth rate of aggregate output and wages incomes and the growth rate of consumption, we use the market-clearing condition (18) to note that the growth equation for consumption, $c_{t+1} = (1 + g_c) \; c_t$, translates into a second-order difference equation for aggregate output,

$$n_{t+1} - n_{t+2}K(\hat{q}) = (1 + g_c)(n_t - n_{t+1}K(\hat{q})). \tag{32}$$

The solutions to this difference equation take the general form

$$n_t = N \; (1 + g_c)^t + M \; \frac{1}{K(\hat{q})^t}, \tag{33}$$

where N and M are constants, to be determined by boundary conditions. We claim that in any steady-state equilibrium we must have $N > 0$ and $M = 0$ so that n_t grows at the same rate g_c as the household's consumption. To see this, note that (26) - more concretely, the zero-profit condition - implies $(1 + \hat{r})K(\hat{q}) < 1$, hence

$$\frac{1}{K(\hat{q})} > 1 + \hat{r} > \beta(1 + \hat{r}) = 1 + g_c. \tag{34}$$

From (18) and (33), we have $c_t = n_t - n_{t+1}K(\hat{q}) = N\ (1+g_c)^t(1-(1+g_c)K(\hat{q}))$ so (34) implies that consumption in any period is positive if and only if $N > 0$. From (33) and (34) we also see that production is positive *for all t* if and only if $M \geq 0$; if M was negative, then for any sufficiently large t, the negative term $M/K(\hat{q})^t$ in (33) would outweigh the positive term $N\ (1+g_c)^t$. Finally, we observe that M cannot be positive; if it was, the discounted present value (5) of the household's wage incomes would be unbounded as we compute, using (30), (33) and the positivity of N,

$$w_1L_1 + \sum_{t=2}^{\infty}\prod_{i=1}^{t-1}\left(\frac{1}{1+r_i}\right)w_tL_t = \sum_{t=1}^{\infty}\frac{w_tL_t}{(1+\hat{r})^{t-1}}$$
$$= \delta(\hat{q})\ \sum_{t=1}^{\infty}\frac{n_t}{(1+\hat{r})^{t-1}} > \delta(\hat{q})\ \sum_{t=1}^{\infty}\frac{M}{(1+\hat{r})^{t-1}K(\hat{q})^t},$$

which diverges if $M > 0$; as mentioned above, unboundedness of the discounted present value (5) of the household's wage incomes is incompatible with the existence of an optimal strategy $\{c_t, L_t, B_t^d\}$ for the household and hence with the equilibrium condition (E1)

Given that output and wage incomes grow at the same rate g_c as the household's consumption, we have

$$w_{t+1}L_{t+1} = (1 + g_c)w_tL_t \tag{35}$$

for all t. Given that the wage *rate* w_t grows at the rate $\hat{q}$, this yields the employment dynamics

$$L_{t+1} = \frac{1+g_c}{1+\hat{q}}L_t, \tag{36}$$

i.e., *if* $g_c < \hat{q}$, *employment is shrinking at the constant rate* $\frac{\hat{q}-g_c}{1+\hat{q}}$; *if* $g_c = \hat{q}$, *employment is constant.*[11]

We summarize our findings in the following theorem.

[11] For the standard Solow growth model with exogenous growth of labor productivity

Theorem 1 *For any wage policy* $\{\bar{w}_t\}$ *of the form (24), there exist suitable values of the initial data* $B_0, A_1, n_1,$ *and* $\bar{w}_1 > 0$*, so that the economy has a steady-state equilibrium.*

If $K'(0) < \beta$*, there exists a unique* $g^* > 0$ *that solves (29). If* $g_{\bar{w}} \leq g^*$*, then in any steady-state equilibrium, productivity, consumption, aggregate output, and wages grow at the common, constant rate* g^**, the real interest rate and the employment level are constant. If* $g_{\bar{w}} < g^*$*, any steady-state equilibrium involves full employment in all periods; if* $g_{\bar{w}} = g^*$*, at least one steady-state equilibrium involves full employment in all periods.*

If $K'(0) < \beta$ *and* $g_{\bar{w}} > g^*$*, then in any steady-state equilibrium, productivity and wage rates grow at the rate* $g_{\bar{w}}$*, consumption, aggregate output, and wage incomes grow at a common, constant rate* $g_c < g^*$*, the interest rate is again constant, but employment shrinks at the constant rate* $\frac{\hat{q}-g_c}{1+\hat{q}}$*. The steady-state equilibrium interest rate as well as the common growth rate* g_c *of consumption, aggregate output, and wage incomes are the lower the higher is* $g_{\bar{w}}$*. If* $K'(0) \geq \beta$*, these conclusions are again true with zero in the place of* g^**.*

The proof of Theorem 1 is given in Appendix 5.1. For $g_{\bar{w}} < g^*$, the theorem is little more than an extension of the result in Hellwig and Irmen (1999) asserting that when prices adjust freely to clear all markets, with $K'(0) < \beta$, the economy has a unique equilibrium and, moreover, this equilibrium involves growth at a constant rate $g^* > 0$. Incentives for investments in productivity improvements are provided by the competitive quasi-rents that firms earn during the one period when they have property rights over their innovations. Compatibility of equilibrium productivity growth with permanent full employment is ensured by the real interest rate taking the value r^* at which the household's intertemporal optimization provides for a growth in consumption demand which just balances the growth in potential output as labor becomes more productive.

The most interesting part of the theorem concerns the case $g_{\bar{w}} > g^*$. For this case, we find that *steady-state equilibrium real wages grow at the same rate as productivity, the share of wages in income is constant at* $\delta(g_{\bar{w}})$*, and yet employment is constantly shrinking.* By conventional reasoning, based on static models with exogenous technologies, we should expect employment to

and an exogenous savings rate a similar result is obtained by Funk (1997). Assuming a binding minimum wage that grows at the same rate as the productivity of labor, he finds that capital accumulation is insufficient to maintain employment and employment contracts at a rate equal to the difference between the growth rates of labor productivity and capital. Similar results are obtained by Dabricki and Takayama (1982) and Ramser (1997).

be constant because the increases in real wages that occur are fully compensated by increases in productivity. In fact the detrimental effects of minimum wages on employment become worse as time goes on; employment becomes ever smaller, converging asymptotically to zero.

The underlying mechanism transcends the usual partial-equilibrium effects of real wages on the demand for labor. As discussed in the introduction, with free entry and exit of firms, i.e., with constant returns to scale in aggregate technologies, profit-maximization alone is not even sufficient to determine the overall demand for labor in any period. A central role is played by aggregate goods demand.

In a system with constant returns to scale, changes in the price of one input relative to output must be compensated by changes in the prices of other inputs relative to output, or else production will cease altogether. In the present context, this means that real interest rates must be low if wage policies are aggressive. Given this adaptation of interest rates to wage policies, the set of active firms is indeterminate and so are aggregate output and employment. In contrast to the traditional partial-equilibrium account, the most immediate effects of aggressive wage policies involve adjustments in prices of inputs other than labor rather than in employment.

The evolution of output over time depends on the evolution of aggregate goods demand over time. This evolution in turn depends on the market participants' intertemporal optimization and hence on intertemporal prices. If real interest rates are low, desired consumption growth will be low and hence the growth of aggregate goods demand and output will be low; if the growth rate of desired consumption and hence of aggregate goods demand is too low relative to the growth rate of wages and productivity, the difference feeds into a decline of employment. The economy "runs out of jobs" because distorted intertemporal prices induce demand growth to fall short of productivity growth.

Given that interest rates cannot fulfil their usual allocative role, some of the needed coordination is provided by rationing in the labor market and the mechanism linking output to household income. To see this, note that, by standard arguments, the household's optimal consumption strategy in a steady-state equilibrium with constant interest rate $\hat{r}$ and zero aggregate profits has the closed-loop representation

$$c_t = (1-\beta)\left[\sum_{i=0}^{\infty} \frac{w_{t+i}L_{t+i}}{(1+\hat{r})^i} + B_{t-1}^d\right] \tag{37}$$

for $t = 2, 3, ...$ With $w_{t+i} = \bar{w}_{t+i} = \bar{w}_t(1+g_{\bar{w}})^i$, $L_{t+i} = \hat{L}_{t+i} = \hat{L}_t(1+\lambda)^i$, and $B_{t-1}^d = (1+\hat{r})n_t K(g_{\bar{w}}) = (1+\hat{r})A_t\hat{L}_t K(g_{\bar{w}})$ for all t and i and some λ, this becomes

$$c_t = (1-\beta)\left[\bar{w}_t \hat{L}_t \frac{1+\hat{r}}{(1+g_{\bar{w}})(1+\lambda)} + (1+\hat{r})A_t\hat{L}_t K(g_{\bar{w}})\right], \qquad (38)$$

which grows at the steady-state rate g_c if and only if the quantity constraint on employment $\hat{L}_t$ shrinks at the rate $\frac{g_{\bar{w}}-g_c}{1+g_{\bar{w}}}$.

The role of rationing in the labor market in this analysis is rather more essential than in a traditional static model with constant-returns to scale technologies and exogenously given minimum wages. As is well-known (see, e.g., Heller and Starr (1979)), such models may have indeterminate Keynesian equilibria because any amount of involuntary unemployment may be justified if with a quantity constraint on employment consumer income is low enough for consumer demand to correspond exactly to the supply generated by employment at the given level. In our intertemporal model, such reasoning yields:

Proposition 2 *Assume that $K'(0) < \beta$ and $g_{\bar{w}} \geq g^*$ or that $K'(0) \geq \beta$. Given the wage policy (24), let $\{w_t, r_t\}$, $\{c_t, L_t, B_t^d, n_t, q_t, l_t\}$, $\{\widehat{L}_t, \Pi_t, A_t\}$ be a steady-state equilibrium for initial data $B_0, A_1, n_1, \bar{w}_1$ and assume that $w_1 = \bar{w}_1$. Then for any $\alpha \in (0,1)$, $\{w_t, r_t\}$,$\{\alpha c_t, \alpha L_t, \alpha B_t^d, \alpha n_t, q_t, l_t\}$, $\{\alpha\widehat{L}_t, \alpha\Pi_t, A_t\}$ is a steady-state equilibrium for the wage policy (24) and the initial data $\alpha B_0, A_1, \alpha n_1, \bar{w}_1$.*

The proof of Proposition 2 is trivial and is left to the reader.[12] Proposition 2 is *not* an indeterminacy result: The parameter α, which indexes the different steady-state equilibria concerns initial conditions as well as the equilibrium allocation; indeed for *given* initial conditions is at most one steady-state equilibrium. Moreover the multiplicity of steady-state equilibria in Proposition 2 concerns the *level* of the time paths of economic activities; it does *not* concern their intertemporal structures. All steady-state growth rates are determinate, indeed unique; rationing in the labor market follows the time path it does because this is necessary to provide for consistency of utility maximization of the household and profit maximization of the firms with rationality of their expectations about future prices, wages and quantity constraints on employment.

[12] Proposition 2 encompasses the case $w_t = \bar{w}_t = \delta(g^*)A_t$ for all t; providing a dynamic version of Heller and Starr's unemployment at Walrasian wages. As in a static model, this is possible because, when the constraint $w_t \geq \bar{w}_t$ is binding, unemployment - at Walrasian or any other wages - does not generate any corrective force.

If the economy at any one date t were to "reconsider" the equilibrium it is following, people would find that the sequel is very much determined by the data $A_t = A_{t-1}(1 + q_t)$ and n_t, aggregate capacity output and labor productivity that have already been determined in the previous period; these data fix maximum employment for date t at n_t/A_t, so if the aggregate capacity output that has been provided for is too low relative to labor productivity, *any* continuation from period t on must necessarily involve rationing in the labor market. In terms of short-term causality, for $g_{\bar{w}} > g^*$, the steady-state equilibria exhibited in Theorem 1 involve involuntary unemployment in any one period because investment in the preceding period has not created enough jobs. This in turn is due to the fact that an anticipation of high real wages has induced firms to invest in labor-saving innovations; moreover the anticipation of high real wages has depressed the real rate of interest, and at the low real rate of interest, household savings were insufficient to finance the aggregate investment that would have been needed for subsequent full employment.

4 Extensions and Comments

4.1 Policy Considerations

What can be said about the welfare properties of the steady-state equilibria in Theorem 1? In Hellwig and Irmen (1999), we showed that the laissez-faire growth rate g^* is less than optimal because firms choosing their innovation investments fail to take account of the knowledge spillovers by which current innovations feed into subsequent economy-wide productivity levels. This might lead one to suspect that wage growth at a rate exceeding the laissez-faire rate could be advantageous as it induces higher productivity growth. However with $g_{\bar{w}} > g^*$, the additional productivity growth merely serves to displace labor; it does not induce higher consumption growth. Therefore, the steady-state equilibria with $g_{\bar{w}} > g^*$ are actually dominated by the steady-state equilibrium with growth rate g^* that arises under laissez-faire or, equivalently, a wage policy with minimum wages that are never binding. In welfare terms, wage growth at a rate $g_{\bar{w}} > g^*$ is strictly detrimental.

To see this, note that the household's overall utility of consumption in a steady-state equilibrium with productivity growth rate $\hat{q}$ and consumption growth rate g_c is given as

$$\sum_{t=1}^{\infty} \beta^t \left[\ln c_1 + \ln(1+g_c)^{t-1}\right]$$

$$= \frac{\beta}{1-\beta}\left[\ln(Y_1 - Y_1(1+g_c)K(\hat{q})) + \frac{\beta}{1-\beta}\ln(1+g_c)\right], \tag{39}$$

where $Y_1 = \min(n_1, A_1 L)$ is aggregate output and $Y_1(1+g_c)K(\hat{q})$ is aggregate investment at date 1. For $\hat{q} > g^*$, (39) is less than

$$\frac{\beta}{1-\beta}\left[\ln Y_1 + \ln(1-(1+g_c)K(g^*)) + \frac{\beta}{1-\beta}\ln(1+g_c)\right], \tag{40}$$

which is easily seen to be increasing in g_c as long as $(1+g_c)K(g^*) < \beta$. Given that g^* solves equation (29), $(1+g^*)K(g^*) < \beta$, so for $g_c < g^*$, (40) is less than

$$\frac{\beta}{1-\beta}\left[\ln Y_1 + \ln(1-(1+g^*)K(g^*)) + \frac{\beta}{1-\beta}\ln(1+g^*)\right], \tag{41}$$

the value of the household's overall utility of consumption in the laissez-faire steady-state equilibrium with common growth rate of wages, productivity and consumption equal to g^*. Whereas the laissez-faire growth rate g^* is too low, the use of wage policy to raise the growth rate is counterproductive: *wage growth at a rate exceeding g^* raises productivity growth, but lowers consumption growth.*

If the behavior of wage-setting institutions is taken as given, what other policy measures can be used to counteract the detrimental effects of wage growth? One possibility would be to have an employment subsidy that would compensate firms for the difference between the minimum wage and the laissez-faire equilibrium wage. However, in a world of excessive wage *growth*, such an employment subsidy itself would have to grow over time. Indeed as time goes on, the share of wages covered by the subsidy would asymptotically converge to one.

As an alternative we consider an interest subsidy, which we take to be financed by a tax on the household's labor income. (Given that the household's labor supply is inelastic, the latter is equivalent to a lump sum tax.) With an interest subsidy rate σ, the first-order condition (27) for the household's intertemporal consumption choice becomes

$$\frac{c_{t+1}}{c_t} = \beta(h(\hat{q})^{-1} + \sigma), \tag{42}$$

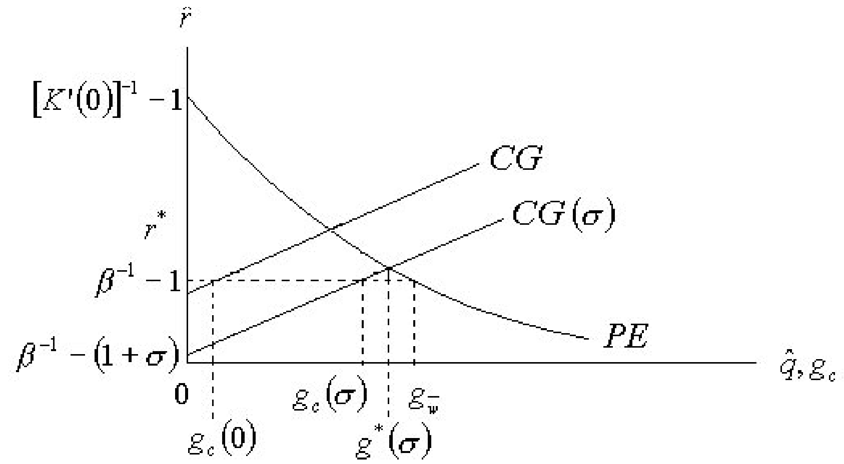

Figure 4: Interest rate subsidy and consumption growth in a steady state equilibrium with $g_{\bar{w}} = \widehat{q} > g_c$.

so the interest subsidy provides a handle on the equilibrium growth rate of consumption. As indicated in Figure 4, in terms of the interest rate paid by producers, the subsidy σ shifts the CG-curve of our Figures 2 and 3 downward. This moves the point of intersection of the CG- and PE-curves to the right. It also raises the consumption growth rate g_c that is compatible with real wage growth and productivity growth at a rate $g_{\bar{w}}$ that exceeds even the rate $g^*(\sigma) > g^*$ that is induced by the subsidy σ in the absence of any wage policy.

In Hellwig and Irmen (1999), we had shown that in the absence of any wage policy, it was actually desirable to use such an interest subsidy to raise the common growth rate of productivity and consumption to the value $g^{**} > g^*$, which is given by the first-order condition

$$K(g_c) + (1-\beta)(1+g_c)K'(g_c) = \frac{\beta}{1+g_c} \tag{43}$$

for the maximization of (39) under the constraint $\hat{q} = g_c$. An interest subsidy that induces the growth rate g^{**} will internalize the knowledge spillovers associated with the innovative activities of firms. If the wage policy involves a growth rate of minimum real wages below g^{**}, the minimum wages would cease to be binding, and the problems caused by excessive growth of minimum wages would be eliminated at the same time.

What about the case $g_{\bar{w}} > g^{**}$? In this case it will be desirable to have an interest subsidy that pushes the equilibrium growth rate of consumption even higher, *above* g^{**}, i.e., the problems caused by excessive wage growth provide an *additional* argument for such an interest subsidy. To see this, note that the same arguments as in Section 3 yield the steady-state conditions

$$\hat{q} = g_w = \max(g_{\bar{w}}, g_c) \tag{44}$$

For consumption growth rates g_c strictly below $g_{\bar{w}}$, this yields $\hat{q} = g_{\bar{w}}$, so in (39), $K(\hat{q}) = K(g_{\bar{w}})$, regardless of g_c. By inspection of (39), this implies that for $g_c < g_{\bar{w}}$, an increase in g_c - and hence an increase in the interest subsidy rate σ - is desirable as long as

$$K(g_{\bar{w}}) < \frac{\beta}{1 + g_c}, \tag{45}$$

which is certainly true at the point $g_c = g^{**}$. The logic underlying this observation is the same as that underlying the first-order condition (43) for optimal consumption growth when the wage policy plays no role: The right-hand sides of (43) and (45) represent the household's marginal rates of substitution for consumption in successive periods, the left-hand sides represent the corresponding rates of transformation in real resource use so both represent a standard trade-off between foregone consumption in one period and additional consumption in the next period. However the marginal rates of transformation on the left-hand sides of (45) and (43) differ because for $g_c < g_{\bar{w}}$, the innovation investment per firm depends only on the growth rate of wages and is (locally) independent of the consumption growth rate g_c. When there is full employment of labor, an increase in consumption growth from one period to the next is feasible only if it is accompanied by a corresponding increase in the rate of productivity growth between the two periods (coupled with a decrease in the subsequent period's rate of productivity growth as firms then can profit from enhanced knowledge spillovers). When there is less than full employment of labor, an increase in consumption growth from one period to the next requires only an increase in the growth rate of the measure of active firms, i.e., of aggregate output.

As for the *optimal* interest subsidy for the case $g_{\bar{w}} > g^{**}$, we must distinguish two constellations: First, if the inequality (45) holds for $g_c = g_{\bar{w}}$, it is optimal to implement this very consumption growth rate, i.e., to fix the interest rate subsidy so as to equate the growth rates of consumption and wages: If g_c were less than $g_{\bar{w}}$, it would satisfy (45) and a small increase in g_c would be desirable. If g_c were greater than $g_{\bar{w}}$, then by (44), we should have $\hat{q} = g_c > g_{\bar{w}}$ and, since $g_{\bar{w}} > g^{**}$, a small decrease in g_c would be desirable.

Second, if the inequality (45) does *not* hold for $g_c = g_{\bar{w}}$, it is optimal to implement the consumption growth rate $g_c = \hat{g}(g_{\bar{w}}) < g_{\bar{w}}$, which satisfies the first-order condition

$$K(g_{\bar{w}}) = \frac{\beta}{1+g_c} \tag{46}$$

for the maximization of (39) when $\hat{q} = g_{\bar{w}} > g_c$. In this case, in contrast to the first one, optimal fiscal policy does *not* preserve full employment of labor. Wage growth - and with it productivity growth - are so fast that the maintenance of full employment of labor would require a high rate of saving and investment. In principle the interest subsidy could be adapted to yield such a rate, but this would not be desirable because at the margin the additional future consumption would not be worth the sacrifice in terms of foregone current consumption.

We summarize our findings in

Proposition 3 *Suppose that the wage policy with growth rate $g_{\bar{w}}$ is taken as given. If $g_{\bar{w}} \leq g^{**}$, it is optimal to have an interest subsidy rate $\sigma^{**} = (1+g^{**})\beta^{-1} - h(g^{**})^{-1}$, inducing consumption growth at the rate g^{**}. If $g_{\bar{w}} \in (g^{**}, \hat{g}]$, where $(1+\hat{g})K(\hat{g}) = \beta$, it is optimal to have an interest subsidy rate $\sigma(g_{\bar{w}}) = (1+g_{\bar{w}})\beta^{-1} - h(g_{\bar{w}})^{-1}$, inducing consumption growth at the rate $g_{\bar{w}}$. If $g_{\bar{w}} > \hat{g}$, it is optimal to have an interest subsidy at the rate $\sigma(g_{\bar{w}}) = (1+\hat{g}(g_{\bar{w}}))\beta^{-1} - h(g_{\bar{w}})^{-1} = K(g_w)^{-1} - h(g_{\bar{w}})^{-1}$, inducing consumption growth at the rate $\hat{g}(g_{\bar{w}})$. For $g_{\bar{w}} \leq \hat{g}$, the optimal interest subsidy induces full employment in all periods. For $g_{\bar{w}} > \hat{g}$, the optimal interest subsidy induces employment contracting with the factor $\frac{1+\hat{g}(g_{\bar{w}})}{1+g_{\bar{w}}}$.*

4.2 Implications for Empirical Research

Our analysis has important implications for the empirical study of the relation between wage policies and employment. First, it shows that changes in labor's share in income should *not* be treated as reliable indicators of wage policy. In the steady-state equilibria of Section 3, the labor share is constant and yet, with $g_{\bar{w}} > g^*$, employment is forever contracting, and unemployment is forever going up. Contrary to what the constancy of the labor share might seem to indicate, the wage policy is *not* employment-neutral.

If we compare steady-state equilibria and postulate, e.g., an innovation cost function of the form $K(q) = kq$, for some $k \in (0, \beta)$, we obtain, from (31) evaluated at $\hat{q} = g_{\bar{w}}$, the steady-state equilibrium labor share as

$$\delta(g_{\bar{w}}) = \frac{1+g_{\bar{w}}}{1+2g_{\bar{w}}}, \tag{47}$$

which is *decreasing* in $g_{\bar{w}}$. When the innovation cost function is linear, a policy that induces higher wage growth involves a lower steady-state equilibrium labor share; this is due to the need to cover the high innovation costs required to keep up with wage growth. For other cost specifications the relation between $g_{\bar{w}}$ and $\delta(g_{\bar{w}})$ may be nonmonotonic,[13] but quite generally it is true that $\delta(0) = 1$ and that $\delta(g_{\bar{w}})$ is *decreasing* in $g_{\bar{w}}$ for $g_{\bar{w}}$ close to zero. Beyond this, no unambiguous general statement about the relation between $g_{\bar{w}}$ and $\delta(g_{\bar{w}})$ seems to be available.

The unreliability of the labor share as an indicator of wage policy is in principle well known. Even in a simple static model, an increase in the real wage rate may *lower* the labor share as firms substitute away from labor. In practice though this possibility is often neglected, mainly because the elasticity of substitution between labor and other inputs is presumed to be less than one. Such a presumption would seem to underlie, e.g., Blanchard's (1999) reference to labor shares as indicators of wage policies in Europe.

However in a dynamic setting one must distinguish between short-run and long-run elasticities of substitution. In our model there is no substitution in the short run because labor is the only current input into production and, e.g., at date 1, the input requirement is fixed at $1/A_1$ units of labor for one unit of output. As long as $w_1 < A_1$, any attempt to estimate short-run substitution behavior will yield an estimated elasticity of substitution equal to zero.

In the long run though there is significant substitution as firms use innovation as a means to reduce labor requirements. From the perspective of period $t-1$, the firm that wants to produce output at t can substitute labor inputs at t by innovation investments at $t-1$. From this perspective the elasticity of substitution between labor and innovation investments is greater than zero; whether it is large enough for the relation between the real wage rate $\bar{w}_t$ and the labor share δ_t to be reversed depends on the innovation cost function. For the linear specification $K(q) \equiv kq$ the elasticity of substitution between labor and innovation investments is easily found to be indeed greater than one and the equilibrium labor share δ_t is *decreasing* in $\bar{w}_t$ whenever $q_t > 0$. More generally substitution may require time, long-run elasticities are larger than short-run elasticities, and the relation between the wage rate and the labor share may depend on the time span that one allows for substitution to take place.

When we say that an increase in the minimum-wage rate $\bar{w}_t$ for date t induces firms to substitute against labor by innovating between dates $t-1$

[13] For example, the specification $K(q) = e^q - 1$ yields $\delta(g_{\bar{w}}) = \frac{1+g_{\bar{w}}}{2+g_{\bar{w}}-e^{-g_{\bar{w}}}}$, which is first decreasing and then increasing in $g_{\bar{w}}$.

and t, we are really referring to the firms' *expectations* at $t-1$ about the wage rates that will prevail at date t. If actual developments and expectations do not coincide, one may have qualms about treating the innovation investment that occurs as a result of actual wage policies. However the parties concerned have strong incentives to try and match expectations to actual developments as best they can, so if there is any systematic structure to the determination of actual wage policies, it seems reasonable to assume that firms exploit this structure as they form their expectations. This is the point of the rational-expectations hypothesis linking expectations to actual developments.

A rational-expectations specification linking endogenous technical change to anticipated future wages provides an alternative approach to the explanation of European developments in the first half of the eighties. In this approach, the decline in labor shares and the increases in capital intensity and labor productivity that occurred at this time (see, e.g., Blanchard 1999) would reflect two sides of the same coin - substitution away from labor.[14] The accompanying increase in unemployment reflected a scarcity of "jobs" resulting from insufficiencies of prior investment (installed capital) as well as aggregate demand. The insufficiency of prior investment was the more significant as production techniques had become relatively more capital-intensive. The "Keynesian" interpretation that, e.g. Blanchard (1999) and Drèze and Bean (1990), attach to these developments fits well into the wider picture we draw provided that the nature of labor productivity growth and the insufficiencies of prior investment (installed capital) and aggregate demand are seen as results of previous developments, driven by expectations, rather than autonomous, period-by-period shocks.

This interpretation is admittedly impressionistic, calling for empirical work rather than presuming upon its results. A serious empirical investigation would have to pay attention to important aspects of macroeconomic developments that we have neglected, most importantly the roles of monetary policy, energy prices, foreign trade, and international capital markets.

Our results also throw some doubt on the practice of treating the *level* of unemployment (in absolute numbers or as a percentage of the labor force) as a relevant variable in empirical macroeconomics. In the steady-state equilibria of Section 3, the relevant variable would be the *rate of change* of employment. To the extent that the forces we analyze are present in the real world, we should expect to see problems in regressions attempting to relate the *level* of unemployment to the growth rate of the economy.[15] Such regressions may

[14] For a related interpretation linking the nature of capital investment to wage policies, see Caballero and Hammour (1997).

[15] Outside of the domain of our analysis, similar problems should be expected for regressions involving the level of unemployment and the inflation rate, e.g. attempts to estimate

run into additional problems if one fails to distinguish between productivity growth and output growth; recall that in our analysis high productivity growth induced by high wage growth will go along with low output growth - and employment contracting at a high rate.

4.3 Non-Steady-State Equilibria

Most of the analysis of this paper has focussed on equilibrium growth rates of productivity and consumption and on the relation between these growth rates. This may seem like an abuse of steady-state analysis. Outside of steady states, neither productivity nor consumption - nor aggregate output - will have constant growth rates. One may therefore wonder what becomes of an analysis based on comparisons of steady-state growth rates when one looks at non-steady-state equilibria.

A systematic analysis of non-steady-state equilibria will be presented in subsequent work. Such equilibria do not, in general, have outcomes converging to steady states. Depending on the data of the model, a non-steady-state equilibrium may involve a stable steady state, a limit cycle of any periodicity, or indeed ergodic chaos.

However the thrust of our analysis does not depend on the restriction to steady states. Our characterization of the equilibrium growth rates of productivity, consumption and employment can be extended to non-steady-state equilibria if we interpret this characterization in terms of long-run averages rather than developments in any one period. More precisely, we find that the geometric means

$$\left(\prod_1^T (1+q_{t+1})\right)^{\frac{1}{T}}, \left(\prod_1^T \frac{c_{t+1}}{c_t}\right)^{\frac{1}{T}}, \left(\prod_1^T \frac{\hat{L}_{t+1}}{\hat{L}_t}\right)^{\frac{1}{T}} \tag{48}$$

of the growth factors $A_{t+1}/A_t = 1 + q_{t+1}$, c_{t+1}/c_t, $\hat{L}_{t+1}/\hat{L}_t$ of productivity, consumption, and employment over T periods $t = 1, 2, ..., T$, converge to well defined limits $1 + g_A$, $1 + g_c$, $1 + g_{\hat{L}}$ as the horizon T becomes large. Moreover, the asymptotic average productivity growth rate g_A coincides with the maximum of $g_{\bar{w}}$, the growth rate of minimum real wages, and g^*, the equilibrium growth rate of real wages under laissez-faire. The asymptotic average growth rate of employment satisfies the relation

$$1 + g_{\hat{L}} = \frac{1+g_c}{1+g_A}, \tag{49}$$

a stable NAIRU, i.e. a rate of unemployment that is compatible with non-accelerating inflation.

so if $g_{\bar{w}}$ exceeds the laissez-faire rate g^*, employment is contracting at an asymptotic average rate that is approximately equal to the difference between $g_{\bar{w}}$ and the induced asymptotic average consumption growth rate g_c. A sketch of the argument is given in Appendix 5.2.

In non-steady-state equilibria as in steady-state equilibria, output growth must in the long-run be equal to demand growth, and employment growth must in the long run correspond to the difference between consumption growth and productivity growth, which in turn is equal to the difference between consumption growth and wage growth. If minimum wages grow at too high a rate this induces an ongoing contraction of employment as productivity growth keeps apace with wage growth and runs ahead of growth in aggregate demand.

4.4 Endogenization of Wage Policies

To conclude the paper, we return to the issue of exogeneity versus endogeneity of wage policies. The specification of an exogenously given wage policy $\{\bar{w}_0(1+g_{\bar{w}})^t\}$ is clearly counterfactual, introduced for the purpose of studying the impact of wage growth on the economy without any presumption that such a policy will actually be followed. If a wage policy of this form with $g_{\bar{w}} > g^*$ was actually followed for some time, we should expect the ensuing decline in employment to induce a change in wage policies, if necessary, through a change in labor market institutions and regulations.

However what change is to be expected? Unless one assumes that in such a situation, the power to impose minimum wages is altogether eliminated, one needs to specify a new policy. The formulation of this policy by the interested parties will reflect these parties views of what is the relation between wages and employment or unemployment. Suppose for example that a union is told that if it equates real-wage growth and productivity growth employment will be unchanged. If productivity growth is estimated with a one-period lag, it may then set $\bar{w}_t = \bar{w}_{t-1}(1 + q_{t-1})$, where q_{t-1} is the innovation rate between periods $t-2$ and $t-1$. Under this wage policy, one has $\bar{w}_t/A_{t-1} = \bar{w}_{t-1}/A_{t-2}$ because $\bar{w}_t$ and A_{t-1} *both* exceed $\bar{w}_{t-2}$ and A_{t-2} by the same factor $(1+q_{t-1})$. But then a straightforward calculation based on (E3) and (E4) shows that the innovation rates q_{t-1} and q_t must be the same, as q_τ for any τ is determined by a time-invariant function of $\bar{w}_\tau/A_{\tau-1}$. Then the innovation rate q_t and with it the growth rate of real wages are constant over time, determined solely by initial conditions. In this way a steady-state equilibrium of the sort that we considered in Section 3 may result from a simple form of endogenizing wages, relying on the traditional benchmark for an employment-neutral wage policy.

Admittedly this endogenization is not based on optimization under rational expectations. The problem is that in the context of our model, with endogenous technical change, the views on the effects of wage policies that are traded in actual policy debate are not actually compatible with rational expectations. Indeed in our model the union official who agrees to a real-wage growth rate less than observed productivity growth may well find that unemployment is still going up and reenter the political arena with the dictum that conventional views on the relation between wages and unemployment are obviously refuted by experience and therefore he could not be bothered to follow conventional prescriptions in the future.

5 Appendix

5.1 Proof of Theorem 1

Assume first that $K'(0) < \beta$. Existence, uniqueness, and positiveness of a solution $\hat{q} = g^*$ to (29) follows from the monotonicity and continuity of the map $\hat{q} \to (1+\hat{q})^2 K'(\hat{q}) + (1+\hat{q})K(\hat{q})$. Given this $g^* > 0$, set

$$q_t = \hat{q} = \max(g^*, g_{\bar{w}}),\ t = 2, 3, ...,$$

$$r_t = \hat{r} = \frac{1}{(1+\hat{q})K'(\hat{q}) + K(\hat{q})} - 1,\ t = 1, 2, ...,$$

$$w_t = \delta(\hat{q})\ A_1\ (1+\hat{q})^{t-1},\ t = 1, 2, ...,$$

$$g_c = \beta(1+\hat{r}) - 1,$$

$$c_t = n_1[1 - (1+g_c)K(\hat{q})](1+g_c)^t,\ t = 1, 2, ...,$$

$$L_t = \hat{L}_t = L\left(\frac{1+g_c}{1+\hat{q}}\right)^{t-1},\ t = 1, 2, ...,$$

$$B_t^d = (1+\hat{r})\ n_1 K(\hat{q})\ (1+g_c)^t,\ t = 1, 2, ...,$$

$$n_t = n_1(1+g_c)^{t-1},\ t = 1, 2, ...,$$

$$l_t = \frac{1}{A_1(1+\hat{q})^{t-1}},\ t = 1, 2, ...,$$

$$\Pi_t = 0,\ t = 2, 3, ...,$$

and

$$A_t = A_1(1+\hat{q})^{t-1}, t = 2, 3, ...$$

It is easy to verify that if the initial data B_0, A_1, n_1, and $\bar{w}_1$ take values so that B_0 is small, $A_1 L = n_1$, and $\bar{w}_1 = \delta(\hat{q})A_1$, then these conditions specify a steady-state Keynes-Radner equilibrium. Notice that this equilibrium involves full employment in all periods if $g_{\bar{w}} \leq g^*$; in this case, the given specification yields $\hat{q} = g^*, \hat{r} = r^*$, $g_c = g^*$, and $L_t = \hat{L}_t = L$ for all t.

It remains to be shown that *all* steady-state equilibria exhibit the features asserted in the theorem. The arguments in the text have already established that in any steady-state equilibrium it must be the case that

- productivity and real wage rates grow at the same rate $\hat{q} \geq g_{\bar{w}}$,
- the interest rate is constant and satisfies (26),
- consumption, aggregate output, and wage incomes have the same, constant growth rate g_c satisfying (28),
- employment is constant if $\hat{q} = g_c$, and shrinks at the constant rate $\frac{\hat{q}-g_c}{1+\hat{q}}$ if $\hat{q} > g_c$,
- $\hat{q} \geq g_c$, with equality only if $\hat{q} = g_c = g^* \geq g_{\bar{w}}$.

It remains to be shown that if $g_{\bar{w}} \leq g^*$, then $\hat{q} = g_c$, if $g_{\bar{w}} < g^*$, then $L_t = L$ for all t, and if $g_{\bar{w}} > g^*$, then $\hat{q} = g_{\bar{w}}$. For the first implication, note that if $\hat{q} > g_c$, then, by (36), $L_t < L$ for $t = 2, 3, ...$, hence, by (K6), $w_t = \bar{w}_t$ for all t, which in turn implies $\hat{q} = g_{\bar{w}}$, hence $g_{\bar{w}} > g_c$. To have $g_{\bar{w}} \leq g^*$ and $\hat{q} > g_c$ *jointly*, would thus require that $g^* \geq \hat{q} = g_{\bar{w}} > g_c$, and therefore, by (26) and (28), that $\hat{r} \geq r^*$ and $\hat{r} < r^*$, which is impossible. Therefore $g_{\bar{w}} \leq g^*$ must imply $\hat{q} = g_c$. Similarly, if $g_{\bar{w}} > g^*$, then $\hat{q} = g_c = g^*$ is impossible, and one must have $\hat{q} > g_c$, hence $\hat{q} = g_{\bar{w}}$ and $g_{\bar{w}} > g_c$. Finally, if $g_{\bar{w}} < g^*$, then $g_{\bar{w}} < \hat{q}$, and one must have $\bar{w}_t < w_t$ and, by (K6), $L_t = L$ for all t. This completes the proof of Theorem 1.

5.2 Non-Steady-State Equilibria: A Sketch of the Argument

Without going into any details, we sketch the argument for the claim made in the text for the simple case when $g_{\bar{w}}$ is large enough so that the restriction $w_t \geq \bar{w}_t$ is always binding. In this case, the set of conditions for an equilibrium can again be decomposed in such a way that equilibrium productivity growth is independent of the demand side of the economy. Exploiting conditions (E3) and (E4) as before, we obtain the following generalizations of (25) and (26).

$$\frac{w_t}{A_{t-1}} \leq \frac{(1+q_t)^2 K'(q_t)}{(1+q_t)K'(q_t) + K(q_t)}, \tag{50}$$

$$r_{t-1} \geq \frac{1}{(1+q_t)K'(q_t) + K(q_t)} - 1 \tag{51}$$

where either inequality is strict only if $q_t = 0$.

One easily verifies that the right-hand side of (50) is a strictly increasing function $f(.)$ of q_t. It follows that for any value of the ratio w_t/A_{t-1}, (50) determines a unique innovation rate

$$q_t = \varphi(\frac{w_t}{A_{t-1}}), \tag{52}$$

where $\varphi(w_t/A_{t-1}) := 0$ if $w_t/A_{t-1} \leq f(0) = 1$, and $\varphi(w_t/A_{t-1}) := f^{-1}(w_t/A_{t-1})$ if $w_t/A_{t-1} > f(0) = 1$.

Consider the implications of wage growth and productivity growth for the evolution of the share $\delta_t := w_t/A_t$ of wages in income. With $w_t = \bar{w}_t = (1+g_{\bar{w}})\bar{w}_{t-1} = (1+g_{\bar{w}})w_{t-1}$ and $A_t = A_{t-1}(1+q_t)$, we have $w_t/A_{t-1} = \delta_{t-1}(1+g_{\bar{w}})$, so the evolution of δ_t is given as

$$\delta_t = \frac{w_{t-1}(1+g_{\bar{w}})}{A_{t-1}(1+q_t)} = \delta_{t-1}\,\frac{1+g_{\bar{w}}}{1+\varphi(\delta_{t-1}(1+g_{\bar{w}}))}. \tag{53}$$

Taking $\delta_1 = w_1/A_1 = \bar{w}_1/A_1$ as given by initial conditions, we find that in any equilibrium in which the constraint $w_t \geq \bar{w}_t$ is binding for all t, the evolution of the share of wages in income is uniquely determined by the difference equation (53). The sequence $\{\delta_t\}$ that is given by (53) and the initial condition $\delta_1 = \bar{w}_1/A_1$ in turn determines the sequence $\{q_t\} = \{\varphi(\delta_{t-1}(1+g_{\bar{w}}))\}$ of equilibrium innovation rates.

From (53), it is obvious that $\delta_t = \delta_{t-1}$ if and only if $q_t = \varphi(\delta_{t-1}(1+g_{\bar{w}})) = g_{\bar{w}}$, or, in view of the definition of $\varphi(.)$, if and only if $\delta_{t-1} = \delta(g_{\bar{w}})$, where, in accordance with (31),

$$\delta(g_{\bar{w}}) = \frac{(1+g_{\bar{w}})K'(g_{\bar{w}})}{h(g_{\bar{w}})} = \frac{(1+g_{\bar{w}})K'(g_{\bar{w}})}{(1+g_{\bar{w}})K'(g_{\bar{w}})+K(g_{\bar{w}})}. \tag{54}$$

As one might have expected from Theorem 1, the evolution of the equilibrium share of wages in income has a unique steady state. However there is no guarantee that this steady state is stable. For instance, if the innovation cost function $K(.)$ is linear, e.g., $K(q) \equiv kq$, $\beta > k > 0$, one finds that the steady state is globally stable if $g_{\bar{w}}$ is sufficiently large, e.g., if $g_{\bar{w}} = .7$, but the difference equation (53) generates ergodic chaos if $g_{\bar{w}}$ is sufficiently small, e.g., $g_{\bar{w}} = .1$.

However, regardless of the stability properties of the sequence of labor shares, the frequency distribution of δ_t over time converges weakly to an ergodic distribution as the horizon over which this frequency distribution is computed becomes large. By a standard ergodic argument, one finds that for given initial conditions there exists a unique distribution F on the unit interval (which may depend on initial conditions) such that for any continuous and bounded function α from $[0,1]$ into $\Re$, the time averages $\frac{1}{T}\sum_1^T \alpha(\delta_t)$ converge to the expectation $\int \alpha(\delta)dF(\delta)$ of $\alpha(.)$ with respect to the distribution $F(.)$ as T becomes large. This has two implications:

- As T becomes large, the time averages

$$\frac{1}{T}\sum_{1}^{T}\ln(1+q_{t+1})=\frac{1}{T}\sum_{1}^{T}\ln(1+\varphi(\delta_t(1+g_{\bar{w}}))) \tag{55}$$

converge to the expectation $\int\ln(1+\varphi(\delta(1+g_{\bar{w}}))dF(\delta)$ of $\ln(1+\varphi(\delta(1+g_{\bar{w}})))$ with respect to the ergodic distribution $F(.)$. This implies that the geometric means $\left(\prod_1^T(1+q_{t+1})\right)^{\frac{1}{T}}$ of productivity growth factors in (48) converge to the fixed quantity $1+g_A:=\exp\int\ln(1+\varphi(\delta(1+g_{\bar{w}})))dF(\delta)$.

- As T becomes large, the time averages

$$\frac{1}{T}\sum_{1}^{T}[\ln\beta-\ln h(\varphi(\delta_t(1+g_{\bar{w}})))] \tag{56}$$

converge to the expectation $\ln\beta-\int\ln h(\varphi(\delta(1+g_{\bar{w}})))dF(\delta)$ of $\ln\beta-\ln h(\varphi(\delta(1+g_{\bar{w}})))$ with respect to the ergodic distribution $F(.)$. For any equilibrium in which (51) holds as an equality for all t,[16] this implies that the geometric means $\left(\prod_1^T\frac{c_{t+1}}{c_t}\right)^{\frac{1}{T}}=\beta\left(\prod_1^T(1+r_t)\right)^{\frac{1}{T}}$ of consumption growth factors in (48) converge to the fixed quantity $1+g_c:=\beta\exp\int\ln h(\varphi(\delta(1+g_{\bar{w}})))dF(\delta)$.

Equality of the asymptotic average productivity growth rate g_A with the minimum-wage growth rate $g_{\bar{w}}$ (when the constraint $w_t\geq\bar{w}_t$ is always binding) follows from the observation that, by (53) and (55),

$$\frac{1}{T}\sum_{1}^{T}\ln(1+q_{t+1})=\frac{1}{T}[\ln\delta_1-\ln\delta_{T+1}]+\ln(1+g_{\bar{w}}) \tag{57}$$

for all T, and that the first term on the right-hand side of (57) vanishes when T becomes large.[17] As for equation (49), this requires showing that in any

[16] Such equilibria always exist, even if equilibrium innovation rates are zero for some periods. However, when equilibrium innovation rates are zero for some periods, there is a continuum of equilibrium allocations of consumption and employment, each of them supported by a different value of the real rate of interest. If geometric time averages of consumption growth factors in these equilibria have well-defined limits, the claims in the text are still valid.

[17] This presumes that δ_t is bounded away from zero. To establish this, note that, by inspection of (50) and (53), $\delta_t\leq 1$ for all t. Since φ is nondecreasing, this implies that

equilibrium the asymptotic average growth factor of output is necessarily equal to $1+g_c$, the asymptotic average growth factor of consumption. For a non-steady-state equilibrium, the proof of this assertion is quite involved, but the underlying economic logic is the same as in the corresponding argument for steady-state equilibria. Details will be presented in subsequent work.

$q_t \leq \bar{q} := \varphi(1+g_{\bar{w}})$ for all t. By inspection of (53), this in turn implies that if $\delta_t \leq (1+\bar{q})^{-1}$ for some t, then for the same t, $\delta_{t-1} \leq (1+g_{\bar{w}})^{-1}$. By (50) though, $\delta_{t-1} \leq (1+g_{\bar{w}})^{-1}$ implies $q_t = 0$, hence $\delta_{t-1} = \delta_t(1+g_{\bar{w}})^{-1} < (1+\bar{q})^{-1}$. Proceeding by induction, one finds that $\delta_t \leq (1+\bar{q})^{-1}$ for some t implies $\delta_{t-k} = \delta_t(1+g_{\bar{w}})^{-k}$ and hence, that $\delta_1 = \delta_t(1+g_{\bar{w}})^{-(t-1)} \leq (1+\bar{q})^{-1}(1+g_{\bar{w}})^{-(t-1)}$, which cannot be true for more than finitely many t.

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