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**MATCHING AND LEARNING IN CITIES:
URBAN DENSITY AND THE RATE OF INVENTION***

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**MATCHING AND LEARNING IN CITIES:
URBAN DENSITY AND THE RATE OF INVENTION***

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This paper examines the role local labor markets play in the production of innovations. We appeal to a labor market matching model (à la Berliant, Reed, and Wang 2004) to argue that in dense urban areas, workers are more selective in their matches and are therefore more productive. We find that, all else equal, *patent intensity* (patents per capita) is 20 percent higher in a metropolitan area with an *employment density* (jobs per square mile) twice that of another metropolitan area. Since local employment density doubles nearly four times across our sample, the implied gains in inventive output are substantial. In addition, we find evidence of an optimal employment density, i.e., one that maximizes patent intensity, of about 2,150 jobs per square mile—roughly the level of Baltimore or Philadelphia. We also find that, all else equal, a city with a more competitive market structure, or one that is not too large (a population less than 1 million) will have a higher patent intensity. These findings confirm the widely held view that the nation’s densest locations play an important role in creating the flow of ideas that generate innovation and growth.

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I. INTRODUCTION

With the emergence of endogenous growth theory in the 1980s, externalities associated with knowledge spillovers have played a prominent role in thinking about sustained economic growth of nations (Romer 1986, Lucas, 1988, and Porter, 1990). Lucas (1988) argues that if these externalities are important, they are most likely manifested in cities that, because of their dense concentration of people and jobs, are best suited to exploit them.

A number of studies attempt to document the existence and significance of localized knowledge spillovers (we review the literature in section II). Many of these rely on a “black-box” model, applying an economy-wide knowledge production function to spatial data. Some recent work has explored the relationship between city size, productivity, and inventive potential, but the relationship between employment density and invention remains largely unexamined.

We believe the inventive output of cities is explained in part by the productivity of worker interactions within firms. As an example, we point to the labor market search model of Berliant, Reed, and Wang (2004), hereafter BRW, which suggests a micro foundation for these externalities. Dense urban agglomeration facilitates invention because workers and firms are more selective about their matches, so the resulting matches are on average more productive. This follows from the lower opportunity cost of rejecting a marginal match—the amount of time and income forgone in the search for a superior partner. Not only are matches more productive, but workers spend a larger share of their time in productive matches. We employ a generalization of their model as the basis for our empirical work.

We explicitly examine the role of employment density on the rate of innovation across metropolitan areas. We use the average rate of patenting per capita—what we call *patent intensity*—in a metropolitan area as a measure of innovations in these areas. We find a

statistically significant relationship between patent intensity and employment density in the highly urbanized portion of metropolitan areas. All else equal, patent intensity is about 20 percent higher in a metropolitan area with employment density that is twice that of another metropolitan area. Since employment density doubles almost four times in our data set, the implied gains in patent intensity are substantial.

We have assembled a very rich data set, which permits us to test a number of related hypotheses. For example, based on the criterion of maximizing patent intensity, we find evidence of an optimal city *size*—about the size of Austin, TX, and optimal employment *density*—about the density of Baltimore or Philadelphia. We find that cities with a more competitive local market structure generate more patents per capita. We also find that our main results are not sensitive to the measure of employment density used—we obtain similar coefficients using all jobs or just certain categories of jobs most likely to consist of knowledge workers. We did not find a significant effect of industrial concentration or diversification in our regressions.

The remainder of the paper is organized as follows. Section II presents a brief review of the existing literature. Section III offers a sketch of our generalization of the BRW labor market matching model (a complete exposition is available in a separate appendix). Section IV describes our data and regression strategy. Section V presents our main results, while Section VI refines our measures of employment density to focus on the role of knowledge workers. Section VII explores econometric issues, including reverse causation, endogeneity, and spatial dependence. Section VIII concludes.

II. THE LITERATURE

The theoretical literature on urban agglomeration economies has focused on three types

of microeconomic foundations: sharing, matching, and knowledge spillovers.¹ The first of these points to the sharing of indivisible factors of production, or the benefits of increased variety of differentiated inputs that occurs in areas with a large number of final-goods producers. The second suggests that larger urban agglomerations facilitate the number and quality of matches among firms and workers. The third argues that the geographic concentration of people and jobs in cities facilitates the local production and diffusion of knowledge.

To date, the bulk of empirical work has focused on testing the more static theories of agglomeration economies.² Less empirical research has been devoted to testing the more dynamic theories associated with knowledge spillovers. Yet Rosenthal and Strange (2004) argue that knowledge spillovers “may be the most interesting of the micro foundations, speaking as they do to so many different areas of economics, including growth theory and the economics of human capital.” As Glaeser (1996) has pointed out, the idea that “growth hinges on the movement of ideas naturally led to a re-exploration of the economic role of cities in furthering intellectual flows.”

While a full review of the literature on the geographic extent of knowledge spillovers is beyond the scope of this paper, we will touch on a few relevant papers.³ In their analysis of U.S. patent data, Jaffe, Trajtenberg, and Henderson (1993, hereafter JTH) find that a new patent is five to 10 times more likely to cite earlier patents from the same city than one would expect based on a control group of other patents.⁴ Rosenthal and Strange (2001) consider the

¹ These themes are developed in the excellent survey by Duranton and Puga (2004).

² Recent surveys of the empirical literature on urban agglomeration economies include Eberts and McMillen (1999) and Rosenthal and Strange (2004).

³ See Audretsch and Feldman (2004) for an extensive review of the literature examining the geographic dimensions of knowledge spillovers.

⁴ But Thompson and Fox-Kean (2003) use a control group of patents selected under different criteria and do not find any evidence of localized knowledge spillovers in patent citations.

importance of input sharing, matching, and knowledge spillovers for manufacturing firms at the state, county, and zip code levels of geography. They find the effects of knowledge spillovers on agglomeration of manufacturing firms tend to be quite localized, influencing agglomeration only at the zip code level.

Several papers look for evidence of knowledge spillovers among agents in different regions and countries. Bottazzi and Peri (2003) find that while R&D spending in a given region tends to increase innovative activity in other regions, the extent of the spillovers tend to be small and highly localized. Peri (2004) examines the geographic distribution of patent citations and finds that regional spillovers in citations tend to be relatively small.

City Size and Density. A number of researchers stress the importance of a metropolitan area's size for innovation. Large metropolitan areas have numerous inventors and plenty of R&D activities that are focused on innovations. Several authors find that patent activity increases with metropolitan area size as measured by population or total employment (Feldman and Audretsch (1999), O'Hallachain (1999), and Bettencourt, Lobo and Strumsky (2004)).

To date, only a few papers examine the role of density in producing local innovations. Ciccone and Hall (1996) find that county employment densities help to explain differences in productivity levels across states. In their study of inventor network effects, Strumsky, Lobo, and Fleming (2005), report a positive relationship between the *number* of patents and population density in 331 MSAs.⁵ This may be further evidence of a scale effects described above. Andersson, Burgess, and Lane (2004) show that the correlation between workers' skills (education) and employers' productivity (revenue per worker) at the establishment level is larger in counties with higher population densities. They argue that this is evidence of superior

⁵ But their regressions do not control for industry or technology mix and their density measure uses an over-inclusive measure of land area (see the discussion on p. 5).

matching between workers and firms in more dense labor markets.

The study by Sedgley and Elmslie (2004) is more closely related to this paper. Relying primarily on regressions of patenting per worker at the *state* level, they report positive and statistically significant coefficients for population density and land area. Sedgley and Elmslie also very briefly discuss a regression for 252 MSAs, and again report significant coefficients. Unfortunately, their regressions may suffer from two econometric problems.

First, their measure of density includes land area that is almost entirely rural. For example, their measure of density in an MSA consists of the entire land area of the counties that define an MSA. But almost 90 percent of the 580,000 square miles of land in MSA counties in 1990 was rural in nature.⁶ This is problematic given that knowledge spillovers are typically associated with high-density urban activities (JTH 1993, Rosenthal and Strange 2001). The bias could easily be systematic—using the definition of cities in this paper (see section IV), the urbanized share of MSA land area varies from less than 1 percent in Yuma, AZ, to 65 percent in Stamford, CT. In this paper, we develop a measure of density that uses only the urbanized portion of land in cities.

The second issue is omitted variable bias. The MSA level regressions reported in Sedgley and Elmslie include only one control variable, the relative size of the manufacturing sector in the 1990s.⁷ In this paper, we include a rich set of controls for industry and technology mix, the presence of high technology firms, and the R&D intensity of private, academic, and government institutions. Each of these controls is lagged, to mitigate the potential effects of endogeneity.

⁶ See Census Bureau (1993), Table 11. This problem is only compounded in the state level regressions, which also suffer from the criticism that states, rather than labor markets, are an arbitrary unit of observation.

⁷ Their state level regressions contain some additional controls.

Human Capital Externalities in Cities. A related literature explores the potential for human capital externalities in cities. Glaeser and Mare (2001) argue that workers develop skills through interactions with one another, and dense locations increase the probability of those interactions. Moretti (2004a) finds that a 1.0-percentage-point increase in the share of a city's population with a college degree led to an increase of 0.4 percent in the wages of college graduates; wages also increased for high-school dropouts (1.6 percent) and high-school graduates (1.9 percent). Glaeser and Saiz (2003) report a strong relationship between share of local population with college education and *subsequent* population growth.⁸

Specialization, Diversification, and Local Market Structure. Following Glaeser, et al. (1992), much of the empirical research has focused on the effects of an economy's industrial structure on innovation and growth. Feldman and Audretsch (1999), using data from the U.S. Small Business Administration Innovation Data Base, found evidence supporting the industrial diversity thesis of Jacobs (1969). Glaeser, et al. (1992) studied employment growth between 1956 and 1987 across specific industries within cities. They found that more industrially diversified metropolitan areas grew more rapidly. In contrast, Henderson, Kuncoro, and Turner (1995) examined employment growth rates between 1970 and 1987 in five traditional capital goods industries located in 224 cities. They found that employment growth in these sectors was positively correlated with a high past concentration in the same industry, supporting the industrial concentration, or Marshall-Arrow-Romer (MAR) view.

Economists debate the effects of an area's market structure on the rate of innovation and growth. Chinitz (1961) and Jacobs (1969) argued that the rate of innovations is greater in cities with competitive market structures. Glaeser, et al. (1992) argue that the MAR view implies that

⁸ See Moretti (2004b) for a review of the literature on human capital externalities in cities.

local monopoly may foster innovation because firms in such environments have fewer neighbors who imitate them. The empirical literature tends to favor the Chinitz and Jacobs view over the MAR view. Feldman and Audretsch (1999) find that local competition is more conducive to innovative activity than is local monopoly. Glaeser, et al. (1992) find that local competition is more conducive to city growth than is local monopoly.

III. HOW LOCAL LABOR MARKETS CONTRIBUTE TO INNOVATION

We believe the inventive output of cities is explained in part by the productivity of worker interactions within firms. This intuition is formalized in the labor market search model of BRW (2004). Suppose that workers migrate to a city until their income (plus any consumption amenities) just equals the incremental costs (rents, taxes, etc.) of living there. Income is generated in teams (pairs), whose output depends on the characteristics of the workers matched together. Workers are differentiated in terms of the variety of knowledge they possess.⁹ The most productive matches occur with an intermediate degree of heterogeneity. The results would be essentially the same if we assume the most productive matches occur among the most homogeneous pairs of workers.

Matches do not last forever, so only a proportion of the population is matched at any time. Unmatched workers engage in a search for an acceptable partner. They meet each other at a rate that depends (positively) on their numbers and the efficiency of the meeting technology. The latter may depend, for example, on the time required to travel to meeting places.

Not all meetings result in a match because a worker's type cannot be recognized until after the workers meet. The probability that any given meeting results in a match is decreasing in workers' *selectivity*—the distance of a marginal partner's type from the worker's ideal type.

⁹ For simplicity, BRW assume that workers are distributed uniformly over a circle of unit circumference.

When workers are very selective, they enter into only highly productive matches, but the probability that any given meeting of unmatched workers results in a match will be low. Hence, there is a trade-off between the benefits of selectivity and the time required to meet and match with suitable partners. Workers' selectivity is endogenous; it is determined by the opportunity cost of matching with a marginal partner, thus forgoing the possibility of making a better match in a subsequent meeting. This, in turn, depends on the arrival rate of future meetings and the selectivity of workers at those meetings.

BRW report comparative static results for changes in the efficiency of the meeting technology, the overall productivity of matches, and locational rents or taxes. These results imply that inventions per capita are higher in cities with a larger population mass. Let $A \cdot y(\delta)$ denote the output from a match where δ represents workers' selectivity and A represents other factors of production or conditions that affect the overall productivity of matches in the city. An increase in A raises workers' expected income, inducing additional migration to the city. The increased population of workers reduces the average waiting time between matches, which, in turn, raises the opportunity cost of entering into a marginal match. This induces workers to be more selective, increasing the average output from matches. A similar intuition applies to an increase in the efficiency of the meeting technology or a reduction in taxes and/or rents.¹⁰

The comparative static results of the model suggest two effects of a larger population mass on per capita inventive output. The first is an increase in the average productivity of matches, as described above. The second is an increase in the steady-state share of workers who are matched and producing inventions. This follows from the fact that unmatched workers wait less time, on average, before finding a suitable partner.

¹⁰ In a separate appendix, available from the authors, we present a generalized version of the BRW model and derive these and other comparative static results.

BRW conclude that “our model provides an important testable hypothesis – cities or other economic units with a higher population mass will also have a higher per capita measure of innovative activity (such as patents).” We can state this hypothesis more precisely: holding constant other attributes of a city, including its physical size, having more workers implies more inventions *per capita*.

It is important to note that in BRW’s model, an increase in population mass is equivalent to an increase in both the scale *and* density of the city. Thus, an empirical test of the theory should take into account both of these factors. The open city version of their model suggests that congestion may adversely affect the invention rate, so it is important to explore this possibility in our regressions.

In richer models, the density of the local labor market is also likely to influence the efficiency of the meeting technology perhaps because travel time is shorter or less effort is required to share ideas or even information about a worker’s type. This characterization is in the spirit of Alfred Marshall’s intuition: “so great are the advantages which people following the same skilled trade get from near neighborhood to one another. The mysteries of the trade become no mysteries; but are as it were in the air, and children learn many of them unconsciously.”¹¹

IV. OUR DATA AND REGRESSION STRATEGY

Since data on innovations are not generally available at the local level, we use patents per capita—what we call *patent intensity*—in a metropolitan area as our measure of innovation. This measure has its shortcomings, since some innovations are not patented and patents differ enormously in their economic impact.¹² Nonetheless, patents remain a useful measure of the

¹¹ See p. 352 of Marshall’s 1895 edition. A number of historical examples of such patterns are presented in Carlino, Chatterjee, and Hunt (2001).

¹² For a general discussion of patents as indicators, see Griliches (1990, 1994).

generation of ideas.

We regress patent intensity in a metropolitan area on measures of local employment density, city size, and a variety of control variables. More specifically, the dependent variable in our regressions is the log of patents per capita averaged over the period 1990-99.¹³ We use an average over the 1990s to minimize any effects from year-to-year fluctuations in patent intensity, which could be an issue in smaller metropolitan areas. To mitigate any bias induced by endogeneity or reverse causation, the independent variables are at 1989-90, or roughly beginning-of-the-period values. In section VI, we investigate these potential biases more closely and find little, if any, effect on our results. Before presenting the exact specification, we will describe the variables used in our regressions.¹⁴

The sample consists of 280 metropolitan areas as defined in 1983. For brevity, we refer to these as *MAs*. Included in this sample are 264 metropolitan statistical areas (MSAs) and primary metropolitan statistical areas (PMSAs). To include as many patents as possible in our data set, we grouped 25 component PMSAs into their corresponding nine consolidated metropolitan statistical areas (CMSAs).¹⁵ It was also necessary to group 21 separate MSAs into seven metropolitan areas.¹⁶ This aggregation permits us to include an additional 9,000 patents (6.5 percent of the total) in our regressions. Our main results are not affected if we drop these observations.

¹³ Our patent data are from the USPTO's US Patent Inventor File and the PATSIC99 file. We thank Jim Hirabayashi of the USPTO for his assistance in obtaining and explaining these data.

¹⁴ Details of the construction of our variables may be found in an appendix available from the authors.

¹⁵ They are Chicago-Gary-Kenosha, IL-IN-WI; Cincinnati-Hamilton, OH-KY-IN; Cleveland-Akron-Lorain, OH; Dallas-Fort Worth, TX; Houston-Galveston-Brazoria, TX; Kansas City, KS-MO; Portland-Vancouver, OR-WA; Seattle-Tacoma, WA, and St. Louis-East St. Louis-Alton, MO-IL

¹⁶ These combinations are Denver-Boulder-Greeley, CO; Greenville and Anderson, SC; Los Angeles and Anaheim, CA; Midland and Odessa, TX; New York-Northeastern New Jersey, NY-NJ; Sarasota and Bradenton, FL; and San Francisco and Oakland, CA.

The Patent Data. Patents are assigned to metropolitan areas according to the *residential* address of the first inventor named on the patent.¹⁷ We allocate patents to a county or metropolitan area when we can identify a unique match to either a county or metropolitan area. Patents that cannot be uniquely matched are excluded from our data set. We were able to locate over 581,000 patents granted over the 1990-99 period to inventors living in the U.S. to either a unique county or MA, a match rate of 96 percent. Just over 534,000 (92 percent) of these patents were associated with an urban county.¹⁸

Land Area. By definition, employment density is the number of jobs per square mile of land area. Employment density varies enormously within metropolitan areas. It is typically highest in the central business district (CBD) of an MA's central city and generally falls off as we move away from the CBD. As pointed our earlier (see Section II), the vast majority of the land in MSA counties is in fact rural in nature and there is also considerable variation in the degree to which the counties surrounding a central city are built out.

We use a measure of land area that reflects the interaction of workers in labor markets that are sufficiently dense to call urban—the *urbanized area* (UA) of cities.¹⁹ These are defined as continuously built-up areas with a population of 50,000 or more, comprising at least one *place* and the adjacent densely settled surrounding area with a population density of at least 1,000 per square mile (U.S. Census Bureau, 1994). While UAs often cross county lines, we collected data on urbanized area land in each county and then aggregated this number to the MA level.

¹⁷ In section VII we verify that our results are not sensitive to the choice of the first inventor's address.

¹⁸ We checked the results of our algorithm against the counts of patents by county in the USPTO's publication (2000). In that source, when an exact county match cannot be made, shares of patents are allocated uniformly across the relevant counties. Our MA patent counts are very similar to those generated using the USPTO method, except in a few instances where two MSAs share a common border (e.g., Dallas and Fort Worth).

¹⁹ Mills and Hamilton (1994, p. 6) argue that urbanized areas correspond most closely to the economist's notion of urban areas.

Employment and Density. For our purposes, the ideal measure of jobs and employment density would count only those jobs located in the urbanized area of cities. Unfortunately, such data are generally unavailable. For example, our preferred measure of employment is derived from the BLS survey of payrolls. We also use these data in our measures of MA size and industrial composition.²⁰ The primary advantage of these data is that jobs are reported based on the *place of work* rather than the *place of residence*. The disadvantage is that the data are reported at the county or MSA level, but not for urbanized areas.

The Census Bureau reported a measure of employment in UAs in the 1990 census, but this count is based on a worker's *place of residence*, not his or her place of work. Most workers live and work in the same UA, but a significant share of UA employment includes workers who live outside the UA. For most UAs, a residency-based measure of employment will understate employment density. The degree of understatement varies considerably across MAs.²¹

While we don't have an ideal measure of employment density, we employ two approximations that should bracket the ideal one. Both measures use the same denominator: the sum of the land area lying in the urbanized area portion of the counties that compose an MA. In the numerator of the first measure, we use the sum of all (establishment-based) employment reported for the same counties. We refer to this measure as *MA employment density*. In the numerator of the second measure, we use (residency-based) employment in the *urbanized* area portion of the same counties, as reported in the 1990 census. We refer to this measure as *UA employment density*.

²⁰ Our jobs data were extracted from the 1999 vintage of the BEA's Regional Economic Indicator System (REIS).

²¹ The ratio of residency-based employment in UAs to establishment-based employment in the associated MAs in our data set is 0.58. The ratio varies from as little as 0.24 in Visalia, CA, to as high as 0.91 in Fort Lauderdale, FL.

To the extent that some metropolitan employment occurs outside of urbanized areas, our MA employment density measure will overstate the actual density of jobs in the built-up portion of MAs. We believe the extent of this overstatement is small and this measure is distinctly superior to alternative measures. In 1990 urbanized areas accounted for 87 percent of the non-rural land area of MSAs, 94 percent of the non-rural population, and 95 percent of non-rural employment by place of residence. The latter statistic probably understates the share of jobs located in urbanized areas because, as Glaeser and Kahn (2001) show, MSA employment is more tightly distributed around the central business district than are residents.

In any case, the most likely effect of such measurement error in our regressions would be a negative bias in the coefficient on employment density. That is because we include in our density measure jobs (in “rural” parts of the MA) less likely to be associated with innovation activities. In that sense, any bias works against our hypothesis. To be conservative, however, we also ran our regressions using our alternative measure, UA employment density. In addition, we report regressions where we instrument for each density measure to better control for possible endogeneity bias or measurement error.

Local Market Structure and Industrial Diversification. To investigate the potential effects of local labor market structure on inventive output, we construct a variable similar to one suggested in Glaeser, et al. (1992)—the number of establishments per worker in the metropolitan area. According to this definition, the higher this ratio, the more competitive is the local labor market.²² This variable may capture more than a static sense of industrial structure. If cities, or industries within a city, are experiencing considerable entry or start-up activity, one would expect average establishment size to be smaller.

²² The number of establishments is derived from the 1989 vintage of *County Business Patterns*.

To explore the possible effects of local industrial diversification or specialization, we construct a Herfindahl-Hirshman Index (HHI) of industry employment shares. Specifically, we calculate the sum of the square of MA employment shares, in 1989, accounted for by seven one-digit SIC industries, plus federal civilian jobs, state and local government jobs, and the remainder.²³ Higher values of this index for an MA imply that its economy is more highly specialized.

Local Research Inputs. Given that our regression relies on a cross section, it is important to take into account factors that influence the overall productivity of matches in a city, the coefficient *A* in our earlier theoretical discussion. We include many control variables for this purpose. For example, it is well known that patent propensity varies significantly across industries, so we include in our regressions the shares of total MA employment in manufacturing and eight other industrial sectors.

We also control for the concentration of firms located in *high technology industries*. We do this by calculating the share of patents obtained in an MA for the years 1980-89 owned by firms in research-intensive industries as defined by the Commerce Department's Office of Technology Policy (2001).²⁴ To control for variations in patent propensity by *field of technology*, we computed the shares of patents obtained in each MA during 1980-89 categorized into one of six technology groups as defined in Hall, Jaffe, and Trajtenberg (2001).²⁵

It is especially important to control for local inputs into the R&D process. For example,

²³ The industries are construction; manufacturing; transportation, communications, and public utilities; wholesale trade; retail trade; services; and FIRE (finance, insurance, and real estate). The remainder consists primarily of jobs in the military, agriculture and mining. All industry breakdowns in this paper are based on the 1987 Standard Industrial Classification system.

²⁴ This variable is constructed by matching patent numbers to assignees (firms) in the NBER Patent Citations Data File and obtaining a corresponding four-digit SIC code from Compustat. We were able to match over 141,000 urban patents (41 percent of the total) granted in the 1980s to firms in the 1999 vintage of Compustat.

²⁵ Every patent in our data set was assigned to one of six broad categories (chemical, computer, medical, electrical, mechanical, and all other). We included the shares of the first five categories in our regressions.

Andersson, Quigley, and Wilhelmsson (2005) find evidence that the expansion of the number of university-based researchers in a local labor market is positively associated with an increase in the number of patents granted in that area.²⁶ To account for the relative abundance of local human capital, our regressions include the share of the population (over 25 years of age) with a college degree or more education in 1990. We also control for the influence of having many nearby universities, a possible college town effect, by including the ratio of college enrollment to population in the years 1987-89.

We include three other measures of research inputs in terms of their *intensities*.²⁷ First, we include in our regressions the sum of spending on R&D in science and engineering at local colleges and universities divided by full-time enrollment at colleges and universities in the MA over the years 1987-89. We hope to capture the intensive margin—the R&D resources available to potential researchers.²⁸ Similarly, our regressions include the sum of federal funding at government research laboratories in the MA divided by the number of federal civilian employees in the MA (averaged over the period 1987-89). Finally, we include in our regressions the number of private R&D facilities in 1989 divided by the number of private non-farm establishments.²⁹

Other Control Variables. Does a correlation between patent intensity and employment density reflect an actual difference in inventive activity or, instead, differences in the way firms protect their inventions? Firms might rely more on patenting in dense areas if it is more difficult to maintain trade secrets there than in less dense areas. In that case, greater difficulty in maintaining secrecy, rather than spillovers, might explain our results.

²⁶Anselin, Varga, and Acs (1997) review studies examining localized spillovers from university R&D.

²⁷ Not surprisingly, the *levels* of these inputs are highly correlated with city size.

²⁸ Ideally, we would want to normalize by full-time S&E faculty or graduate students, but these cannot easily be assigned to particular campuses for a number of university systems that account for a significant portion of R&D.

²⁹ Over 1,800 private labs associated with the top 500 R&D performing corporations were geographically located using information contained in the 1989 edition of the *Bowker Directory of American Research and Technology*.

To test the significance of this alternative explanation, we create an index of the importance of trade secrecy that varies across metropolitan areas. We do this by weighting industry-specific measures of the effectiveness of trade secrecy reported in Cohen, Nelson, and Walsh (2000) by the industry shares reflected in the mix of private R&D facilities in every MA in our data set. A higher value of this index for an MA implies that trade secrets are relatively more effective for the mix of industries reflected in its R&D facilities.

We include a number of other control variables. We control for variations in demographics by including the share of the population in 1990 that is of working age. We also include the percent change in employment over the years 1980-89 as a control for the effects of unobserved differences in local economic opportunities on inventive activity. We also include seven dummy variables based on the BEA economic region in which the MA is located (the Rocky Mountain region is omitted).

Our Specification. Our main regression equation is simply:

$$P_i = C + a_1 D_i + a_2 D_i^2 + a_3 E_i + a_4 E_i^2 + a_5 COMP_i + \sum_{g=6}^{14} a_g INDSHR_i + a_{15} HITECH_i + \sum_{k=16}^{20} a_{16k} PATCLASS_{ik} + a_{21} PCTCOL_i + a_{22} CE_i + a_{23} U_i + a_{24} FEDLAB_i + a_{25} R \& D_i + a_{26} TS_i + a_{27} EMPGT_i + a_{28} WAP_i + \sum_{j=29}^{35} a_j REGION_{ij} + \varepsilon_i$$

where:

P_i = Log of average patents per capita, 1990-99 in the i -th MA;

D_i = Log of MA job density in 1989 in i or UA job density in 1990 in i ;

E_i = Log of 1989 level of employment in MA $_i$;

$COMP_i$ = Log of the number of establishments in MA $_i$ divided by total employment in MA $_i$, in 1989;

$INDSHR_i$ = The share of employment in one-digit SIC industries in 1989 MA $_i$;

$HITECH_i$ = Share of patents in MA $_i$ during 1980-89 obtained by firms in R&D intensive industries;

$PATCLASS_{ik}$ = Share of patents obtained in MA $_i$ during 1980-89, classified in one of five technological categories;

$PCTCOL_i$ = Percent of 1990 population over 25 with at least a college degree in MA_i ;
 CE_i = Ratio of the college enrollment to population in i ;
 U_i = University R&D spending per student, averaged for 1987-89 in MA_i ;
 $FEDLAB_i$ = Federal lab R&D per federal civilian job, averaged for 1987-89 in MA_i ;
 $R \& D_i$ = Private lab R&D per establishment in 1989 in i ;
 TS_i = Trade Secrets Index = The log of a weighted average of ratings of the effectiveness of trade secret protection in i ;
 $EMPGT_i$ = The percent change in employment in MA_i during the period 1980-89;
 WAP_i = Share of the working age population in i ;
 $REGION_{ij}$ = dummy variables indicating in which of the eight BEA regions MA_i is located;
 ε_i is the random error term.

V. THE MAIN RESULTS

Table 1 shows the summary statistics for the variables used in the analysis. The average number of patents per 10,000 of population obtained over the 1990s—our measure of patent intensity—is about 2. San Jose stands out, with a patent intensity of 17. At the other end of the distribution, the patent intensity for McAllen, TX, is only 0.07. Figure 1 demonstrates the skewness of patent intensity across cities.

The urbanized land area of MAs varies considerably across cities: For Grand Forks it is less than 15 square miles; for New York-Northeastern New Jersey, it exceeds 3,000 square miles. Establishment-based employment in our MAs varies from 37,000 (Caspar, WY) to 9.6 million (New York-Northeastern New Jersey), while residency-based employment in the urbanized areas of these MAs varies from 17,000 to 7.6 million. The mean of MA employment density is 1,727 jobs per square mile while the mean of UA employment density is 987 jobs per square mile. The latter varies from 263 jobs per square mile (Gadsden, AL) to 2,777 jobs per square mile (Los Angeles-Long Beach).

Figure 2 plots the log of patent intensity against the log of MA employment density. A moderate correlation (0.39) is clearly evident; there is a similar correlation between patent intensity and UA employment density. In the regressions that follow, we explore how much of this correlation remains after controlling for the many other factors that are likely to influence inventive activity. The model is estimated using ordinary least squares in STATA, but we report robust standard errors (White correction) to control for any heteroskedasticity.

Employment Density and City Size. Table 2 presents the main results of the paper. The regressions in columns 1 and 3 show that, however measured, the effect of employment density on patent intensity is positive and statistically significant.³⁰ These coefficients can be interpreted as elasticities. All else equal, patent intensity is about 17 percent to 20 percent higher in an MA that is twice as dense as another MA. Employment density varies by more than 1,200 percent across the sample, so the implied gains in the per capita invention rate are substantial.

Columns 2 and 4 report the results from regressions that add the square of our density measures as independent variables. There is clear evidence of diminishing returns at very high density levels. The optimal level—according to our MA employment density measure—is 2,190 jobs per square mile. That is about the 75th percentile of our data set, about the levels of Baltimore (2,168) and Philadelphia (2,181). In section VI, we explore more narrow definitions of employment density (e.g., scientists and engineers). We again find evidence of an optimal density using these measures, but only at levels attained by about 10 percent of our sample.

We now turn to the question of scale economies in the more traditional sense. Previous research (Feldman and Audretsch (1999) O hUallachain (1999) and Bettencourt, Lobo and Strumsky (2004)) suggests that measures of innovation are positively related to metropolitan size

³⁰ Unless otherwise noted, t statistics are reported in parentheses.

(population). Yet no research to date has considered the offsetting effect of congestion on innovative activity, as implied by the open city model of BRW. When we include MA employment (in logs), but not its square, in our regressions (not shown) the coefficient on this measure of city size is not statistically significant.³¹ When we include the squared term, as we report in Table 2, the coefficients on these variables are statistically significant.

The implied optimal size, measured in terms of MA employment, is about 500,000 jobs, about the 80th percentile of the size distribution in our data. If we assume a labor force participation rate of 66 percent, this corresponds to a population of about 750,000, roughly the size of Austin, TX, or Raleigh-Durham, NC, in 1990. Thus, after controlling for the effects of employment density, the benefits of urban scale are realized for cities of moderate size. In fact, with the exception of San Jose, the top 5 percent of our metropolitan areas ranked in terms of patent intensity had populations below 1 million in 1989.

Local Competition. The regressions suggest that the rate of innovation is enhanced in more competitive local environments characterized by many small firms, rather than in local economies dominated by a few large firms. The coefficient on the number of establishments per employee is about 1.6 and is precisely measured. The coefficient can be interpreted as an elasticity since the variable is included in logs in our regression. The effect is economically significant, as this ratio more than doubles across our sample. This result is consistent with the views of Chinitz (1961), Feldman and Audretsch (1999), Glaeser, et al. (1992), and Jacobs (1969) that competitive local labor markets facilitate innovation. We are not able to determine whether this results from static (market structure) or dynamic (firm entry) effects, or both.

³¹ The coefficient is 0.03 with a p value of about 0.40.

Industrial Mix and Specialization. Patent activity varies enormously across industries. As expected, the manufacturing share of MA employment is positively related to local patent intensity. All else equal, a 10 percent increase in the manufacturing share of employment is associated with a 3 percent increase in patent intensity. Conversely, a 10 percent increase in the state and local government share of employment is associated with a 4.5 percent *decrease* in patent intensity.

If knowledge spillovers occur largely within industries, specialized cities may be more efficient producers of inventions. On the other hand, if important spillovers are generated across industries, perhaps more industrially diverse cities may be more efficient innovators. To test for such effects, we constructed a commonly used measure of concentration, an HHI of industry employment shares (see section IV). When we include this variable in our regressions (not shown), the estimated coefficient is never statistically significant.³² We also constructed a measure of technological specialization using our technology share controls. When we included this variable in our regressions (not shown) the estimated coefficient was negative, but not significant ($p = 0.13$). In short, our results suggest that while the mix of industries is obviously important, the overall concentration or dispersion of economic activity across industries is not.

Local Research Inputs. The results reported in Table 2 clearly show that local research inputs are important to explaining the variation in patent intensity across MAs. The coefficients on our controls for research-intensive industries and the controls for most technology fields are statistically significant and precisely measured. These variables capture characteristics relevant to patent intensity that are not fully explained by local industry mix and structure. The largest elasticities, evaluated at the mean, are for chemical inventions (0.30), mechanical inventions

³² When all our employment shares are also included, the coefficient on HHI is essentially zero. If we only include the manufacturing share of employment, the coefficient on HHI is negative, but is not significant.

(0.24), computers (0.19), and high-technology industries (0.16).

By far the most powerful effect is generated by human capital (the share of the adult population with at least a college degree). A 10 percent increase in this ratio is associated with an 8.6 percent increase in patents per capita. We also included a variable to capture the relative size of higher education in a metropolitan area, measured by the ratio of college enrollment to population. The coefficient on this variable (not shown) is not significant in our regressions, suggesting there is no separate college town effect on the local invention rate.³³

Our other controls for local research intensities include the ratio of academic R&D in science and engineering to student enrollment (in 1987-89), federal lab R&D spending per federal civilian employee (in 1987-89), and the number of private R&D labs per 1,000 establishments (1989). All of these variables have a positive impact on the rate of patenting, but the implied elasticities are relatively small. For example, a 10 percent increase in private R&D intensity is associated with only a 1 percent in patent intensity. The elasticity for academic R&D intensity is slightly smaller (.08). Still, these effects are economically significant because there is considerable variation in academic and private R&D intensity in our data (see Table 1).

Agrawal and Cockburn (2002) argue that local academic R&D is likely more productive, in terms of its contribution to additional patents, in the presence of a large research intensive firm located nearby—the *anchor tenant* hypothesis. Taking this effect into account, they report a significant positive correlation between local patents and academic publications in the fields of medical imaging, neural networks, and signal processing. We looked for a more general interaction—do cities with a relative abundance of academic and private R&D enjoy a disproportionately high patent intensity? We tested for this by interacting our measures of

³³ In other regressions we included the log of the number of colleges and universities in the MA. But the coefficient on this variable is never statistically significant.

academic and private R&D intensity and including them in our regressions (not shown). We were surprised to find a significant, but negative coefficient (-0.13) on this interaction term.³⁴

There does appear to be some degree of substitution between local academic and private R&D investments, but the effect is quite small—the implied elasticity at the mean is -.03.³⁵

Trade Secret Protection. Recall that we constructed an index of the efficacy of trade secret protection among firms located in an MA. If the estimated coefficient on this variable is negative, we might be concerned that firms are substituting patents for trade secret protection in dense areas because the former are relatively more effective in such environments. We find, instead, the estimated coefficient is positive but insignificant at standard confidence levels. This is consistent with Cohen, Nelson, and Walsh (2000), who find a positive correlation in firms' rating of the effectiveness of trade secrecy and patent protection. It is also consistent with the result in Fosfuri and Rønde (2004), who find that trade secret protection stimulates clustering in a model of firm location in the presence of information spillovers. In any case, city size and employment density remain important in explaining patent intensity even after controlling for an industry's reliance on trade secret protection.

Helsley and Strange (2004) argue that knowledge transfers between agents may arise through a form of barter in the absence of established property rights in the underlying ideas. They argue this barter process may be more effective in smaller metropolitan areas where anonymity is harder to maintain. In larger MAs, informal exchange (or cooperation) may become unsustainable and agents are forced to patent their ideas before they can exchange them for

³⁴ The p value is .026. The coefficients on the academic and private R&D variables remain significant; in fact they increase by more than the estimated coefficient on the interaction (but the changes in these coefficients are not statistically significant). The other regression coefficients hardly change. Note that the correlation between the private and academic R&D intensities in our data is only 0.17.

³⁵ We also interacted the R&D intensity of private and government labs, but found no significant effects.

anything valuable. To test this hypothesis, we interacted our trade secrets variable with city size and, alternatively, with employment density (not shown), but we did not find any statistically significant interactions. These results suggest that the phenomenon we are measuring is real, i.e., there really are more inventions.

Employment Growth and Other Control Variables. The coefficient on employment growth in the previous decade (not shown) is positive but not statistically significant in our main regressions (it is sometimes significant in other specifications).³⁶ This is true even when we drop our establishments per worker variable, which might also pick up variations in city or industry dynamics. Our demographic control, the share of the population of working age, is always positive but is statistically significant in only some regressions. The estimated coefficients on two of the seven BEA region dummies (not shown) are statistically significant. MAs located in the New England and Southwest regions had lower patent intensities. Overall, it appears that our controls do a good job of accounting for the other factors that contribute to innovation in cities.

VI. The Density Knowledge Workers

To this point, our measures of employment density reflect the entire workforce of the MA. Not all of these jobs are directly involved in the process of inventing new products or processes. So it is reasonable to ask whether it would be better to instead focus on a measure of occupations consisting of the knowledge workers in an MA.

We avoid doing this in our main regressions (Table 2) for several reasons. First, it is not obvious what the appropriate set of occupations should be. Second, a substantial amount of invention occurs when users of a product or process modify it to suit their particular needs (Morrison, Roberts, and Von Hippel 2000). These users may not fall into the occupations we

³⁶ It varies from about 0.28 to about 0.34 in the regressions reported in Table 2.

might include in the class of knowledge workers. Third, our industry, technology, and human capital controls ought to absorb most of the effect of the unobserved variation in the composition of the workforce. If our general measures impart a bias, then the bias should work against us.

Nevertheless, we re-estimate our specifications using two more narrow measures of employment density. The first includes only those jobs falling into the Census Bureau's classification of professional specialty occupations.³⁷ This grouping includes engineers, scientists, social scientists, doctors, and other health professionals. But it also includes teachers, lawyers, artists, and athletes. The second includes only scientists and engineers living in the urbanized area in 1990.³⁸ Both of these are residency-based measures of employment in 1990.

In Table 3, we report results using each measure in our primary specifications. In the first and third columns of the table, we show that the estimated coefficient on employment density is about 0.22 and is measured very precisely ($p < 0.01$). The estimated coefficients on most other variables change only slightly. The estimated coefficient on our human capital measure falls a bit, especially when we use the density of scientists and engineers in our regressions. The estimated coefficients on manufacturing employment share are also a bit smaller.

We also constructed a density measure counting only jobs that *do not* fall into the Census Bureau's professional and specialty classification. This measure explicitly excludes scientists, engineers, medical professionals, and college professors. Yet, if we include only this measure of density in our regression (not shown), the estimated coefficient is 0.19 and is statistically significant ($p < 0.05$). If we include both density measures in the regression (not shown), professional specialty occupations and those jobs that do not fall in the professional specialty occupations, the coefficient on the latter measure is negative but insignificant, while the

³⁷ This grouping refers to the 1990 standard occupation classification codes 043-202.

³⁸ This measure is constructed from tabulations at the urbanized area level published by the Census Bureau.

coefficient on the former measure rises and remains significant. We conclude that while much of the effect of density on patent intensity is concentrated in these more narrow categories of jobs, using our general measures of job density does not bias our results.

Columns 2 and 4 of Table 3 verify there are diminishing returns to employment density even when using these measures. The optimal density of professional specialists is 320 per square mile, the 88th percentile of our sample. The optimal density of scientists and engineers is 57 per square mile, the 92nd percentile of our sample. Thus, in our data, relatively few MAs exhaust the returns to scale associated with the density of these jobs. The estimated optimal scale, measured in terms of population, in these regressions falls to about 650,000 to 700,000.

VII. TESTING FOR ROBUSTNESS

In this section, we examine a number of factors that might potentially affect our results. We consider alternative specifications, reverse causation and endogeneity bias, and spatial dependence. None of the main results are affected after controlling for these issues.

Alternative Specifications. To this point, we have associated inventions with MAs on the basis of the home address of the first inventor listed on the patent. One might wonder about how the first inventor is selected and whether this process might affect our regression results. For example, suppose a multinational company patents an invention developed by researchers working in separate labs in different cities or even countries.

For a variety of reasons, we do not believe such concerns should significantly affect our results. About 49 percent of our patents have only one inventor. Among the other patents, only 2.6 percent involve inventors living in different countries, and only a third of these report a first inventor living in the U.S. Among the patents where the first two inventors live in an American city, nearly 70 percent live in the same MA. When inventors do live in separate MAs, they tend

to live far apart. The average distance is 560 miles.

Table 4 reports two sets of regressions. The first two columns of Table 4 are based on the same specification reported in columns 1 and 3 of Table 2, except we add 3 new variables to the regressions: the share of the MA's patents with a second inventor residing in another MA, the log of the average distance between inventors' MAs for those patents, and the square of this distance. The coefficient on each of these variables is statistically significant. All else equal, the higher the share of an MA's patents with a second inventor living in another MA, the lower is the MA's patent intensity. This is not surprising as it is likely that firms with a more decentralized workforce are also likely to have a more even spatial distribution of patents. The estimated coefficients on our density measures are somewhat larger than reported in Table 2, while the estimated optimal city size falls to about 500,000.

Somewhat surprisingly, the coefficient on the average distance between inventors' MAs is positive (the coefficient on the square of distance is negative). Conditional on relying on a distant co-inventor, the optimal distance between MAs is 270-330 miles, depending on the density measure used in the regression. These results suggest that inventors may be taking advantage of differentiated knowledge available in other MAs, a finding consistent with the intuition of BRW.

In Columns 3 and 4 of Table 4 we report the findings when we repeat the specification used in columns 1 and 3 of Table 2, except that the observations are based on the address of the *second* inventor on the patents. The coefficients on the density and size variables are statistically significant and take the same sign as in Table 2. Similar results (not shown) are obtained when we use any of our other density measures. We conclude that our findings are not sensitive to the choice of the first inventor's address in our analysis.

Reverse Causation, Endogeneity, and Consumption Amenities. Our regressions estimate the effects of employment density and city size on patent intensity. In this section, we directly address the possibility of reverse causation—patent intensity might affect city size, employment density, or both.

We begin with simple Granger causality tests (not shown). In the *forward* regression, we regress patent intensity in the 1990s on patent intensity in 1975-79, and MA employment density in 1989 (all in log form). The coefficient on the last of these variables is 0.43 and significant at the 1 percent level. In the *reverse* regression, we regress MA employment density over the 1990s on employment density in 1989 and patent intensity in 1975-79 (all in log form). The coefficient on the lag of patent intensity is significant at the 1 percent level, but it is also very small (-0.01). While we reject the hypothesis of no reverse causation, the estimated effect is more than an order of magnitude smaller than the relationships estimated in our main regressions.

Even though all of our independent variables are significantly lagged, one may still be concerned about the possibility of endogeneity and a resulting bias in the estimated coefficients. A related concern is that a correlation between patent intensity and employment density might occur if highly productive (i.e., inventive) workers are attracted to MAs by consumption amenities (e.g., variety) not adequately controlled for in our regressions and which are not already reflected in our human capital variables.³⁹ To address these possibilities, we perform instrumental variables (2SLS) regressions and examine Hausman tests for endogeneity bias. We instrument for employment density, employment, and its square.

In addition to the other right-hand-side variables in our main regressions, we include as

³⁹ While it is possible that such amenities may attract more population, and thus employees, it does not explain the negative correlation between patent intensity and urbanized land area in a regression (not shown) controlling for employment and our other control variables.

instruments a variety of weather and topographic variables. The existence of a significant correlation between such variables and density has been documented in other work (Rappaport 2003). We also include deep lags of MA urbanized land area (1980) and employment (1970), in logs, and the square of these variables. Finally, to address the possibility that other consumption amenities explain our results, we include as instruments the number of museums, restaurants, violent crimes, and property crimes in 1989, each expressed in per capita terms.⁴⁰

Our weather and topography variables are derived from the USDA's Economic Research Service Natural Amenity Scale.⁴¹ These data are reported at the county level and include mean hours of sunlight in January, mean temperature in January and July, and the percent of county area covered by water. These variables are aggregated to MAs, weighting by county land area.⁴² We also construct dummy variables that reflect the presence of five topographic features in MA counties: plains, tablelands, open hills and mountains, hills and mountains, and plains with hills and mountains.

The F statistic in each of the first stage regressions is at least 24 or higher, suggesting that our instruments are strong. Columns 1 and 3 of Table 5 report OLS estimates for the same sample of cities we can estimate using our instruments (we lose six observations owing to missing variables). Columns 2 and 4 report the coefficients from the instrumental variables (IV) regressions. The estimated coefficients on MA employment density fall somewhat relative to OLS in the IV regression but the opposite pattern is observed when we examine the regressions with UA employment density. This is what we would expect when we correct for measurement error in these two variables (see section IV). In any case, Hausman tests do not identify any

⁴⁰ These data are derived from County Business Patterns.

⁴¹ For more information, see <http://www.ers.usda.gov/data/naturalamenities> and McGranahan (1999).

⁴² We also include the water area of MA counties, in square miles, as reported by the Census Bureau for 1990.

systematic differences between the OLS and IV coefficients in these regressions.

We also performed IV regressions using an even deeper lag of urbanized land area (1970) as an instrument (not shown). The estimated coefficients on employment density and city size are slightly larger than the comparable OLS estimates, but they are no longer statistically significant.⁴³ Again, Hausman tests do not identify any systematic differences between the IV and OLS estimates. We conclude that any remaining endogeneity in our regressions is unlikely to explain our main results.

Spatial Dependence. There is a very high degree of spatial inequality in the distribution of patent activity. Patenting tends to be highly concentrated in the metropolitan areas of the northeast corridor, around the Research Triangle in North Carolina, and in California's Silicon Valley. Even though the coefficients on our regional dummy variables are typically insignificant, this clustering of innovative activity suggests there could be strong spatial dependence at a more localized level and, if so, it should be controlled for in our empirical analysis.

The conjecture, then, is that patent intensity in one MA may be highly correlated with patent intensity in nearby MAs. The consequences of spatial autocorrelation are the same as those associated with serial correlation and heteroskedasticity: When the error terms across MAs in our sample are correlated, OLS estimation is unbiased but inefficient. However, if the spatial correlation is due to the direct influence of neighboring MAs, OLS estimation is biased and inefficient (Anselin 1988).

The literature suggests two approaches to dealing with spatial dependence. In the first approach, spatial dependence is modeled as a spatial autoregressive process in the error term:

⁴³ In these regressions, the p values for the coefficients on MA and UA employment density, respectively, are 0.12 and 0.16. The sample size in these regressions is only 227.

$$\begin{aligned}\varepsilon &= \lambda W \varepsilon + \mu \\ \mu &\sim N(0, \sigma^2)\end{aligned}$$

where λ is the spatial autoregressive parameter and μ is the uncorrelated error term. W is a spatial weighting matrix where nonzero off-diagonal elements represent the strength of the potential interaction between the i th and j th MAs. We use the inverse of the square of the geographic distance between MAs to fill in the off-diagonal elements of W . The null hypothesis of no spatial error dependence is $H_0 : \lambda = 0$.

The second approach models the spatial dependence in patenting activity via a spatially “lagged” dependent variable:

$$P = \rho WP + X\beta + \varepsilon$$

where P is an $N \times 1$ vector and N is the number of locations in our study; ρ is the autoregressive parameter (a scalar); W is the $N \times N$ spatial weight matrix described above; X is an $N \times K$ matrix of other explanatory variables from before; and ε is the $N \times 1$ random error term. The null hypothesis of no spatial lag is $H_0 : \rho = 0$.

Following Anselin and Hudak (1992), we perform three tests for spatial autocorrelated errors: Moran’s I test, the Lagrange multiplier (LM) test, and a robust Lagrange multiplier test (robust LM). We also perform two tests for the spatial lag model (LM test and a robust LM test). The Moran’s I test is normally distributed, while the LM tests are distributed χ^2 with k and one degree of freedom, respectively.

We estimate each of the specifications previously reported in Table 2 using these various tests for spatial dependence. The results are summarized in Table 6. The null hypothesis of zero spatial lag cannot be rejected in any specification. The results for spatial error are somewhat more ambiguous. The null hypothesis is clearly rejected according to the Moran’s I test, but not

according to the LM and robust LM tests. Anselin (1990) reports that the Lagrange multiplier tests are more robust than the Moran's I test under Monte Carlo simulations, which suggests that spatial error is unlikely to be an issue for our specifications.

Nevertheless, we re-estimate each specification reported in Table 2, incorporating a correction for either spatial error or spatial lag. Table 7 presents the results for the specifications used in columns 1 and 3 of Table 2.⁴⁴ As expected, we did not find any instances of a significant spatial error or spatial lag coefficient. The primary effect of using maximum likelihood procedures is that most of the coefficients are estimated more precisely.

VIII. CONCLUSION

Patent intensity—the per capita invention rate—is positively related to the density of employment in the highly urbanized portion of MAs. All else equal, the number of inventions per person is about 20 percent greater in an MA with a local economy that is twice as dense as another MA. Since local employment density doubles more than four times in the sample, the implied gains in patents per capita due to urban density are substantial. These results are consistent with theories that suggest that density may contribute to more efficient matching in the labor market (BRW 2004). In short, we find empirical evidence consistent with a theoretical microfoundation of endogenous growth.

In addition, we find evidence of increasing returns to scale in the invention process, but holding density constant, these returns are exhausted at a modest city size—certainly below 1 million in population. Similarly, we find evidence of diminishing returns to density, but only at

⁴⁴ These estimates were obtained using the *Spatreg* procedure in STATA. The results for the specifications reported in columns 2 and 4 of Table 2 are nearly identical to the OLS results.

levels attained by a quarter of our sample.⁴⁵ Both results are consistent with theories of labor market matching that allow for in-migration and, therefore, congestion effects.

Our results strongly support theories that suggest that more competitive local market structures are more conducive to innovation. We find that industrial and technology mix are important in explaining the variation in patent intensity across cities, but we found no significant effects for our measures of industrial or technological specialization. We found that local R&D inputs, especially human capital, contribute to higher patent intensities and there is evidence of a very modest substitution effect between academic and private R&D intensity. Variations in the reliance of a city's industries on trade secret protection firm's did not have a significant effect in our regressions.

A logical extension of this paper is to investigate how the spatial distribution of a firm's inventors and R&D facilities influence its inventive productivity. That is the subject of our ongoing research.

⁴⁵ Diminishing returns to density sets in much later in our sample (about the 90th percentile) if we instead use only on scientists and engineers in our density measure.

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Figure 1: Patent Intensity Across MAs

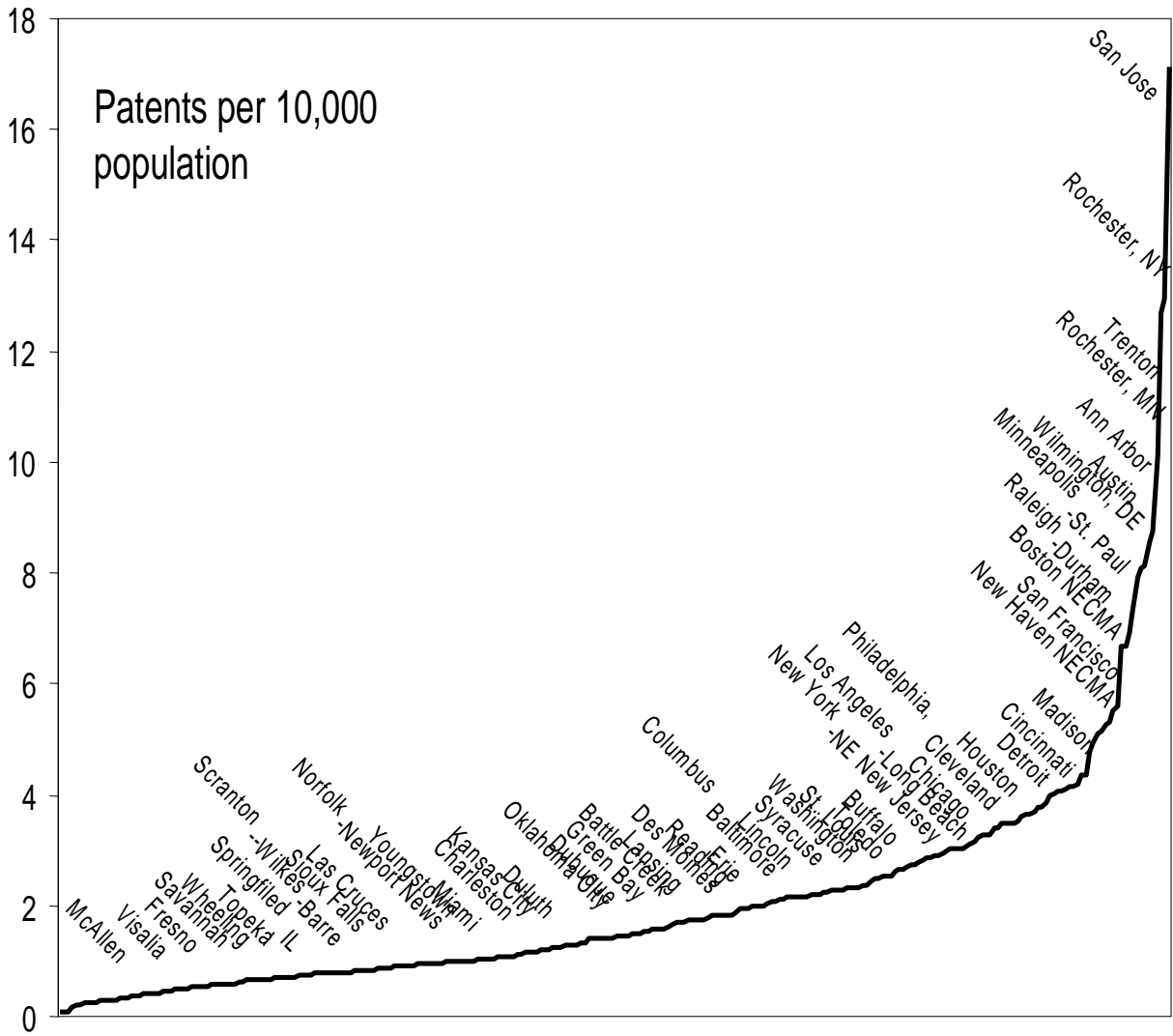


Figure 2: Patents per Capita & MA Employment Density

Log of Patents per
10,000 Population

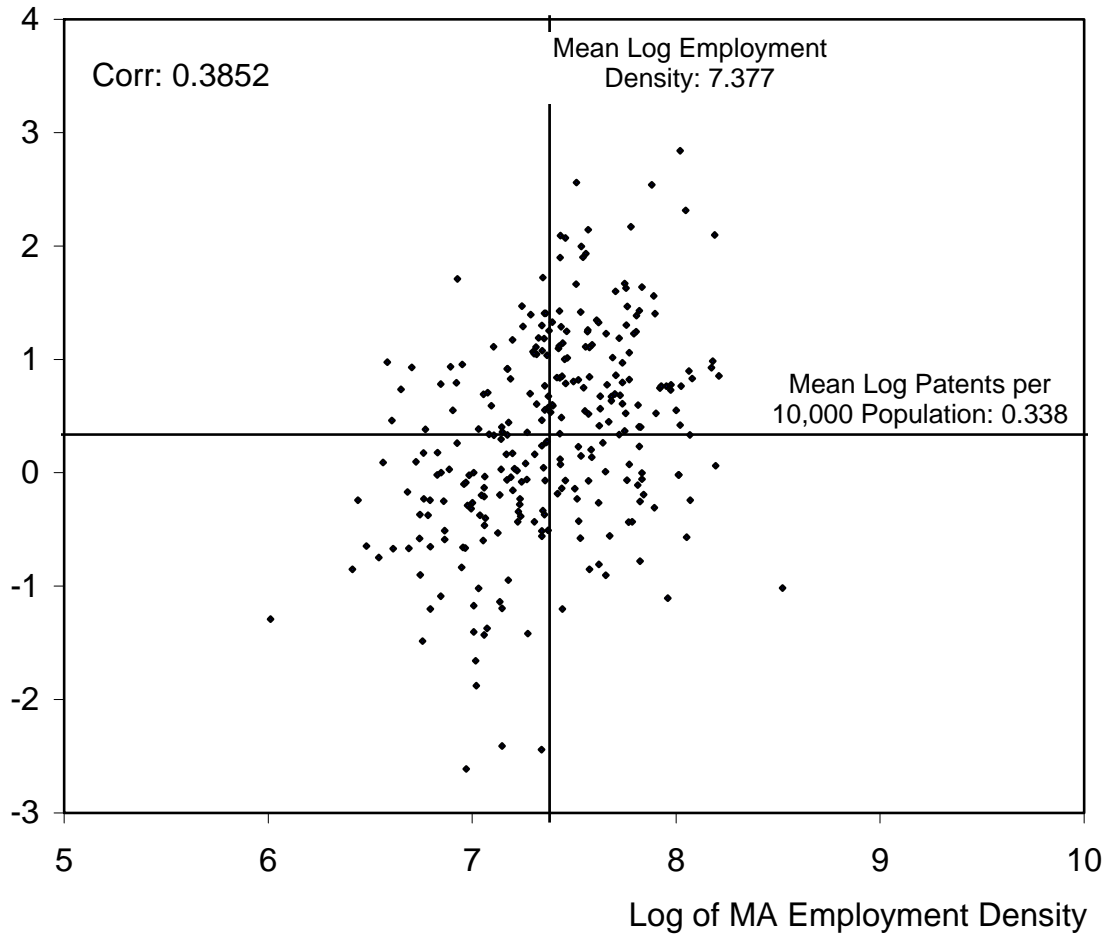


Table 1: Descriptive Statistics				
	Mean	SD	Min.	Max.
Patents Per 10,000 of Population, Avg. 1990-99	2.057	2.110	.0732	17.14
MA Employment Density, 1990	1,727	689.3	408.1	5,021
UA Employment Density, 1990	987.4	405.9	263.4	2,777
Urbanized Area Land Area, 1990	211.5	333.5	14.50	3015
Urbanized Area Land Area, 1980	182.4	298.8	14.00	2808
Urbanized Area Land Area, 1970	155.2	262.4	2.2	2425
MA Water Area, 1990	178.4	373.1	.3700	2483
MA Water Area, 1990 (percent)	7.810	12.58	.0300	65.11
MA Employment, 1989	392,480	862,483	37,375	9,665,015
MA Employment, 1970	320,765	719,959	34,059	8,368,789
UA Employment, 1990	265,431	663,744	17,406	7,563,283
MA Employment Growth, 1979-89 (percent)	20.47	15.54	-25.80	77.69
Working Age Population, 1990 (percent)	64.43	3.077	53.85	74.79
Ratio of UA to MA Employment, 1989-90	.5864	.1350	.2472	.9396
HHI of Industry Employment Shares, 1989	.1791	.0186	.1456	.2819
Establishments per 100,000 Employees, 1989	4425	597.8	2667	6365
Manufacturing Employment, 1989 (percent)	14.92	7.447	1.815	46.06
Construction Employment, 1989 (percent)	5.371	1.305	2.881	11.02
Transportation Employment, 1989 (percent)	4.426	1.522	1.553	11.88
Wholesale Employment, 1989 (percent)	4.349	1.385	.6752	9.178
Retail Employment, 1989 (percent)	17.71	1.935	11.96	24.83
Services Employment, 1989 (percent)	25.79	4.207	9.823	44.78
FIRE Employment, 1989 (percent)	6.729	2.042	2.679	16.68
Federal Civilian Employment, 1989 (percent)	2.297	2.385	.2936	20.84
State & Local Gov. Employment, 1989 (percent)	11.74	4.671	4.405	34.55
High-tech Patents, 1980-89 (percent)	18.78	19.47	0	88.91
Chemicals Patents, 1980-89 (percent)	17.14	12.82	0	76.11
Computer Patents, 1980-89 (percent)	5.692	6.453	0	48.23
Medical Patents, 1980-89 (percent)	9.210	10.55	0	88.00
Electrical Patents, 1980-89 (percent)	6.418	6.089	0	44.80
Mechanical Patents, 1980-89 (percent)	24.89	10.03	5.600	62.37
College Educated, 1990 (percent)	19.54	6.235	8.100	45.40
Enrolled in College, 1987-89 (percent)	6.661	5.423	0	34.06
University R&D Spending (\$1,000) per Student, Avg. 1987-89	.5623	.9324	0	5.297
Federal Lab R&D Spending (\$1,000) per Federal Civilian Employee, 1987-89	1.396	10.81	0	161.4
Private R&D Labs per 1,000 Establishments, 1989	.3037	.3863	0	2.710
Trade Secrets Index	50.96	5.382	34.04	70.69
Restaurants per 10,000 of Population, 1989	16.28	2.676	8.910	29.06
Museums per 10,000 of Population, 1989	.1308	.0917	0	.4806
Violent Crimes per 10,000 of Population, 1989	54.35	30.71	6.604	220.4
Property Crimes per 10,000 of Population, 1989	521.8	165.1	140.7	956.9

Dependent Variable: Patents per Capita †	1	2	3	4
MA Employment Density, 1989 †	0.195 (2.21)**	4.138 (2.24)**		
MA Employment Density Squared †		-0.269 (2.14)**		
UA Employment Density, 1990 †			0.169 (1.70)*	3.847 (1.98)**
UA Employment Density Squared †				-0.273 (1.92)*
MA Employment (10,000), 1989 †	0.407 (2.73)***	0.366 (2.36)**	0.410 (2.74)***	0.317 (1.96)*
MA Employment Squared †	-0.053 (2.90)***	-0.047 (2.45)**	-0.055 (3.02)***	-0.039 (1.92)*
Working Age Population, 1990 (%)	2.441 (1.72)*	2.033 (1.46)	2.236 (1.59)	2.125 (1.52)
Establishments per Employee, 1989 †	1.577 (4.59)***	1.585 (4.69)***	1.592 (4.76)***	1.624 (4.90)***
Manufacturing Employment, 1989 (%)	2.060 (2.99)***	1.947 (2.80)***	2.169 (3.06)***	2.251 (3.10)***
Construction Employment, 1989 (%)	-0.302 (0.10)	-0.006 (0.00)	-0.648 (0.21)	-0.826 (0.27)
Transportation Employment, 1989 (%)	-3.249 (1.22)	-3.396 (1.26)	-3.304 (1.26)	-3.072 (1.19)
Wholesale Employment, 1989 (%)	-2.388 (0.73)	-2.698 (0.84)	-1.586 (0.50)	-1.911 (0.61)
Retail Employment, 1989 (%)	-3.749 (1.69)*	-4.376 (2.02)**	-3.956 (1.80)*	-4.339 (2.01)**
Services Employment, 1989 (%)	0.315 (0.33)	0.041 (0.04)	0.135 (0.14)	0.145 (0.15)
FIRE Employment, 1989 (%)	1.235 (0.68)	1.195 (0.67)	0.996 (0.54)	1.236 (0.69)
Federal Civilian Employment, 1989 (%)	-1.992 (1.37)	-2.380 (1.64)	-2.167 (1.48)	-2.057 (1.44)
State & Local Gov. Employment, 1989 (%)	-3.861 (3.07)***	-3.958 (3.11)***	-3.825 (2.97)***	-3.858 (3.00)***
High-tech Patents, 1980-89 (%)	0.861 (4.42)***	0.820 (4.15)***	0.865 (4.43)***	0.838 (4.29)***
Chemicals Patents, 1980-89 (%)	1.728 (4.31)***	1.749 (4.39)***	1.758 (4.39)***	1.772 (4.44)***
Computer Patents, 1980-89 (%)	3.306 (5.47)***	3.423 (5.70)***	3.332 (5.46)***	3.390 (5.57)***
Medical Patents, 1980-89 (%)	-0.442 (0.72)	-0.336 (0.54)	-0.498 (0.79)	-0.329 (0.51)
Electrical Patents, 1980-89 (%)	0.870 (1.87)*	1.014 (2.21)**	0.878 (1.89)*	0.977 (2.18)**
Mechanical Patents, 1980-89 (%)	0.975 (2.21)**	0.994 (2.23)**	0.949 (2.12)**	0.950 (2.13)**
College Educated, 1990 (%)	4.390 (4.90)***	4.368 (4.92)***	4.434 (4.91)***	4.353 (4.87)***
University R&D per Student, 1987-89	0.143 (3.12)***	0.142 (3.02)***	0.146 (3.18)***	0.146 (3.15)***
Federal Lab R&D / Fed Civ Jobs, 1987-89	0.007 (3.70)***	0.006 (3.62)***	0.006 (3.49)***	0.006 (3.30)***
Private R&D Labs / Establishments, 1989	0.340 (4.56)***	0.336 (4.73)***	0.323 (4.31)***	0.330 (4.66)***
Trade Secrets Index (Lab-weighted) †	0.308 (1.23)	0.326 (1.32)	0.363 (1.41)	0.319 (1.26)
Constant	-18.815 (5.47)***	-32.873 (4.67)***	-18.701 (5.64)***	-30.911 (4.11)***
Adjusted R-squared	0.786	0.789	0.784	0.787

Notes: N = 280. Regressions include a lag of MA employment growth, the share of the population enrolled in college, a constant, and BEA region dummies. † Included in log form in regression.

* significant at 10 percent. ** significant at 5 percent. *** significant at 1 percent.

Table 3: Knowledge Worker Regressions– Robust Standard Errors				
Dependent Variable: Patents per Capita †	1 [#]	2 [#]	3 ^{##}	4 ^{##}
Professional Specialty Jobs Density, 1990 †	0.228 (2.72)***	2.125 (2.18)**		
Professional Specialty Jobs Density Squared †		-0.185 (1.94)*		
Scientists & Engineers Density, 1990 †			0.218 (3.05)***	0.601 (3.14)***
Scientists & Engineers Density Squared †				-0.074 (2.16)**
MA Employment (10,000), 1989 †	0.396 (2.69)***	0.343 (2.23)**	0.343 (2.28)**	0.271 (1.75)*
MA Employment Squared †	-0.053 (2.93)***	-0.045 (2.35)**	-0.048 (2.73)***	-0.036 (1.93)*
Working Age Population, 1990 (%)	2.478 (1.75)*	2.379 (1.70)*	1.624 (1.21)	1.842 (1.35)
Establishments per Employee, 1989 †	1.526 (4.55)***	1.530 (4.61)***	1.468 (4.44)***	1.420 (4.40)***
Manufacturing Employment, 1989 (%)	2.017 (2.95)***	1.842 (2.72)***	1.629 (2.43)**	1.500 (2.23)**
Construction Employment, 1989 (%)	-0.338 (0.11)	-0.080 (0.03)	-1.908 (0.62)	-2.066 (0.67)
Transportation Employment, 1989 (%)	-3.078 (1.17)	-3.391 (1.30)	-4.540 (1.70)*	-4.762 (1.81)*
Wholesale Employment, 1989 (%)	-2.335 (0.73)	-2.836 (0.90)	-0.773 (0.24)	-1.326 (0.43)
Retail Employment, 1989 (%)	-4.025 (1.81)*	-4.772 (2.18)**	-3.859 (1.78)*	-4.393 (2.06)**
Services Employment, 1989 (%)	0.154 (0.16)	-0.099 (0.11)	-0.297 (0.32)	-0.401 (0.43)
FIRE Employment, 1989 (%)	1.298 (0.70)	1.130 (0.62)	0.411 (0.22)	0.322 (0.18)
Federal Civilian Employment, 1989 (%)	-2.259 (1.61)	-2.637 (1.87)*	-3.455 (2.51)**	-3.523 (2.57)**
State & Local Gov. Employment, 1989 (%)	-4.246 (3.38)***	-4.325 (3.41)***	-4.390 (3.31)***	-4.507 (3.43)***
High-tech Patents, 1980-89 (%)	0.834 (4.32)***	0.815 (4.18)***	0.779 (4.18)***	0.800 (4.22)***
Chemicals Patents, 1980-89 (%)	1.706 (4.27)***	1.693 (4.23)***	1.639 (4.08)***	1.616 (4.04)***
Computer Patents, 1980-89 (%)	3.165 (5.30)***	3.257 (5.46)***	2.807 (4.45)***	2.984 (4.67)***
Medical Patents, 1980-89 (%)	-0.440 (0.72)	-0.346 (0.57)	-0.386 (0.64)	-0.403 (0.67)
Electrical Patents, 1980-89 (%)	0.877 (1.91)*	0.964 (2.15)**	0.801 (1.81)*	0.893 (1.99)**
Mechanical Patents, 1980-89 (%)	0.967 (2.21)**	0.965 (2.19)**	0.854 (2.00)**	0.847 (2.02)**
College Educated, 1990 (%)	4.226 (4.73)***	4.316 (4.90)***	3.929 (4.43)***	4.007 (4.56)***
University R&D per Student, 1987-89	0.140 (3.03)***	0.137 (2.98)***	0.136 (3.06)***	0.145 (3.20)***
Federal Lab R&D / Fed Civ Jobs, 1987-89	0.007 (3.70)***	0.006 (3.44)***	0.005 (2.58)**	0.005 (2.55)**
Private R&D Labs / Establishments, 1989	0.330 (4.50)***	0.336 (4.87)***	0.282 (3.81)***	0.341 (4.65)***
Trade Secrets Index (Lab-weighted) †	0.312 (1.26)	0.315 (1.30)	0.376 (1.60)	0.342 (1.47)
Adjusted R-squared	0.788	0.790	0.796	0.798

Notes: Regressions include a lag of MA employment growth, the share of the population enrolled in college, a constant, and BEA region dummies. † Included in log form in regression. # N = 280. ## N = 278.

* significant at 10 percent. ** significant at 5 percent. *** significant at 1 percent.

Table 4: Second Inventor Regressions – Robust Standard Errors

	1	2	3	4
	Patents per Capita †	Patents per Capita †	Patents per Capita, 2 nd Inventor †	Patents per Capita, 2 nd Inventor †
MA Employment Density, 1989 †	0.217 (2.43)**		0.262 (1.90)*	
MA Employment Density, 1990-99 Avg. †				
UA Employment Density, 1989 †		0.264 (2.59)**		0.486 (3.36)***
MA Employment (10,000), 1989 †	0.268 (1.89)*	0.261 (1.85)*	0.578 (3.19)***	0.552 (3.11)***
MA Employment Squared, 1989 †	-0.038 (2.22)***	-0.041 (2.39)**	-0.070 (3.06)***	-0.074 (3.28)***
2 nd Inventors not in Same MA as 1 st Inventor, 1990-99 (%)	-0.537 (3.34)***	-0.600 (3.67)***	-2.518 (8.79)***	-2.634 (8.99)***
Average distance between MA of 1 st and 2 nd Inventor, 1990-99	0.427 (3.98)***	0.451 (4.13)***	0.889 (5.80)***	0.938 (5.90)***
Average distance between MA of 1 st and 2 nd Inventor Squared, 1990-99	-0.038 (3.11)***	-0.041 (3.26)***	-0.098 (5.47)***	-0.103 (5.52)***
Working Age Population, 1990 (%)	2.368 (1.79)*	2.111 (1.61)	4.107 (1.87)*	3.704 (1.69)*
Establishments per Employee, 1989 †	1.542 (5.06)***	1.606 (5.34)***	1.591 (3.60)***	1.759 (3.92)***
College Educated, 1990 (%)	4.225 (5.07)***	4.235 (5.03)***	4.160 (4.08)***	4.081 (4.08)***
Enrolled in College, 1987-89 (%)	-0.609 (0.61)	-0.609 (0.61)	-0.886 (0.64)	-0.963 (0.71)
University R&D per Student, 1987-89	0.155 (3.52)***	0.157 (3.59)***	0.184 (3.27)***	0.184 (3.34)***
Private Labs / Establishments, 1989	0.006 (3.03)***	0.005 (2.90)***	0.009 (2.94)***	0.009 (3.11)***
Federal Lab R&D / Fed Civ Jobs, 1987-89	0.382 (5.17)***	0.360 (4.89)***	0.408 (3.85)***	0.373 (3.55)***
Trade Secrets Index (Lab-weighted) †	0.339 (1.37)	0.418 (1.64)	0.763 (1.98)**	0.899 (2.37)**
Employment Growth, 1980-89 (%)	0.333 (1.52)	0.362 (1.67)*	0.311 (0.96)	0.332 (1.03)
Constant	-19.027 (6.19)***	-19.816 (6.47)***	-24.542 (5.07)***	-27.435 (5.52)***
Adjusted R-squared	0.809	0.810	0.807	0.813

Notes: N = 280. Regressions include lagged industry employment shares, high-tech industry patent share, lagged patent class shares, and BEA region dummies. † Included in log form in regression.

* significant at 10 percent. ** significant at 5 percent. *** significant at 1 percent.

	1	2	3	4
Dependent Variable: Patents per Capita †	OLS	IV	OLS	IV
MA Employment Density, 1989 †	0.228 (2.58)**			
MA Employment Density, 1990-99 Avg. †		0.160 (1.66)*		
UA Employment Density, 1990 †			0.185 (1.83)*	0.218 (1.78)*
MA Employment (10,000), 1989 †	0.403 (2.69)***		0.411 (2.72)***	
MA Employment Squared, 1989 †	-0.053 (2.87)***		-0.056 (3.02)***	
MA Employment (10,000), 1990-99 Avg. †		0.353 (2.20)**		0.355 (2.21)**
MA Employment Squared, 1990-99 Avg. †		-0.047 (2.42)**		-0.050 (2.59)**
Working Age Population, 1990 (%)	2.320 (1.59)	2.280 (1.59)	2.135 (1.48)	2.105 (1.45)
Establishments per Employee, 1989 †	1.530 (4.20)***	1.457 (4.06)***	1.549 (4.36)***	1.509 (4.16)***
College Educated, 1990 (%)	4.579 (4.71)***	4.646 (4.77)***	4.630 (4.72)***	4.613 (4.65)***
Enrolled in College, 1987-89 (%)	0.131 (0.12)	0.036 (0.03)	0.233 (0.22)	0.100 (0.09)
University R&D per Student, 1987-89	0.146 (3.16)***	0.153 (3.27)***	0.149 (3.21)***	0.154 (3.28)***
Private Labs / Establishments, 1989	0.007 (3.84)***	0.006 (3.42)***	0.007 (3.49)***	0.006 (3.45)***
Federal Lab R&D / Fed Civ Jobs, 1987-89	0.336 (4.39)***	0.338 (4.41)***	0.315 (4.10)***	0.316 (4.09)***
Trade Secrets Index (Lab-weighted) †	0.328 (1.30)	0.348 (1.37)	0.388 (1.49)	0.403 (1.55)
Employment Growth, 1980-89 (%)	0.353 (1.44)	0.352 (1.39)	0.379 (1.56)	0.394 (1.58)
Constant	-18.726 (5.18)***	-17.651 (5.05)***	-18.537 (5.33)***	-18.449 (5.08)***
Adjusted R-squared	0.791	0.791	0.788	0.788

Notes: N = 274. In addition to the independent variables used in Table 2, our instruments include temperature in January and July, days of sunlight in January, surface water in square miles and as a share of total area, 5 dummy variables for topography, urbanized land area in 1980 (in logs) and its square, employment in 1970 (in logs) and its square, the number of restaurants and museums in 1989 (in logs) and violent and property crime rates in 1989. The second stage regressions include lagged industry employment shares, high-tech industry patent share, patent class shares, and BEA region dummies.

† Included in log form in regression.

* significant at 10 percent. ** significant at 5 percent. *** significant at 1 percent.

Table 6: Spatial Dependence Tests ^a (P-values)				
Density Measure:	MA Employment Density			
Specification:	Table 2.1	Table 2.2	Table 2.1	Table 2.2
Test for:	Spatial Error		Spatial Lag	
Moran's I $\lambda = 0$	0.000	0.000		
LM - $\lambda = 0$	0.350	0.401		
Robust LM- $\lambda = 0$	0.788	0.749		
LM - $\rho = 0$			0.156	0.254
Robust LM- $\rho = 0$			0.270	0.404
Density Measure:	UA Employment Density			
Specification:	Table 2.3	Table 2.4	Table 2.3	Table 2.4
Test for:	Spatial Error		Spatial Lag	
Moran's I $\lambda = 0$	0.000	0.000		
LM - $\lambda = 0$	0.369	0.306		
Robust LM- $\lambda = 0$	0.835	0.582		
LM - $\rho = 0$			0.151	0.275
Robust LM- $\rho = 0$			0.255	0.504

Notes: N = 280. ^aMoran's I is based on standardized z-values that follow a normal distribution. The Lagrange multiplier (LM) tests are distributed as χ_1^2 with critical levels of 3.84 (p = 0.05).

Table 7: Patent Intensity Regressions—Correcting for Spatial Effects (Robust Standard Errors)				
	1	2	3	4
Dependent Variable: Patents per Capita †	Spatial Error		Spatial Lag	
MA Employment Density, 1989 †	0.206 (2.42)**		0.188 (2.29)**	
UA Employment Density, 1990 †		0.171 (1.81)*		0.158 (1.70)*
MA Employment (10,000), 1989 †	0.410 (2.93)***	0.413 (2.94)***	0.402 (2.88)***	0.406 (2.90)***
MA Employment Squared †	-0.053 (3.11)***	-0.055 (3.23)***	-0.052 (3.09)***	-0.055 (3.20)***
Working Age Population, 1990 (%)	2.635 (1.99)**	2.375 (1.81)*	2.401 (1.81)*	2.207 (1.68)*
Establishments per Employee, 1989 †	1.540 (4.69)***	1.563 (4.88)***	1.577 (4.94)***	1.589 (5.11)***
College Educated, 1990 (%)	4.283 (5.10)***	4.359 (5.14)***	4.443 (5.27)***	4.488 (5.29)***
Enrolled in College, 1987-89 (%)	0.323 (0.34)	0.386 (0.40)	0.269 (0.28)	0.340 (0.35)
University R&D per Student, 1987-89	0.144 (3.41)***	0.147 (3.46)***	0.144 (3.35)***	0.147 (3.41)***
Federal Lab R&D / Fed Civ Jobs, 1987-89	0.006 (3.87)***	0.006 (3.61)***	0.007 (4.03)***	0.006 (3.80)***
Private R&D Labs / Establishments, 1989	0.346 (4.92)***	0.327 (4.63)***	0.335 (4.82)***	0.319 (4.56)***
Trade Secrets Index (Lab-weighted) †	0.344 (1.47)	0.391 (1.62)	0.290 (1.23)	0.342 (1.42)
Employment Growth, 1980-89 (%)	0.273 (1.16)	0.322 (1.39)	0.357 (1.59)	0.386 (1.74)*
Constant	-19.448 (6.01)***	-19.109 (6.14)***	-18.704 (5.84)***	-18.540 (6.02)***
Log Likelihood	-131.28	-132.85	-131.35	-132.66
λ	.0342 (1.02)	.0259 (0.78)		
ρ			.4182 (1.13)	.4205 (1.13)

Notes: N = 280. Z statistic reported in parentheses. Regressions include lagged industry employment shares, high-tech industry patent share, patent class shares, and BEA region dummies.

† Included in log form in regression.

* significant at 10 percent. ** significant at 5 percent. *** significant at 1 percent.

APPENDICES to
WORKING PAPER NO. 04-16/R
MATCHING AND LEARNING IN CITIES:
URBAN DENSITY AND THE RATE OF INVENTION *

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The views expressed here are those of the authors and do not necessarily represent the views of the Federal Reserve Bank of Philadelphia or the Federal Reserve System.

Appendix A: A Model of Matching and Innovation

Introduction

In our paper, we argue that the inventive output of cities may be explained in part by the nature of worker interactions within firms. A formalization of this intuition can be found in the labor market search model of Berliant, Reed, and Wang (2004), hereafter BRW. In this appendix, we develop a more general version of their model and derive a number of comparative static results. We extend their model in several ways. We assume a more general production function for inventions. We allow for consumption amenities that influence the equilibrium size of cities. Finally, we characterize the effect of an exogenous increase in land area on city population and inventive output.

The Model

Time is continuous. Workers live forever and discount consumption at the rate r . A measure N of these workers decides to live in the city because the increase in their income, plus utility derived from the available local public goods exceeds the utility lost to the effects of congestion (including higher rents and taxes). Once located in the city, workers meet randomly and decide whether to match in order to engage in R&D that results in inventions. If the agents agree to match, the match may continue indefinitely, but all matches are subject to a constant dissolution rate π . Workers consume equal shares of the resulting output and their resulting utility is linear in the output consumed.

The output generated in a match depends on the characteristics of the workers matched together. Workers are differentiated in terms of the variety of knowledge they possess. Agents' types are distributed uniformly along a circle of unit circumference. The most productive matches occur with an intermediate degree of heterogeneity. We focus on the properties of the unique symmetric stationary equilibrium of the game.

In steady state, a measure of agents $M \in [0, N]$ will already be matched and a corresponding measure $U = N - M$ will engage in search for a new partner. In a given match, workers produce inventions at the rate $A \cdot y(x)$. A is a scalar that reflects the overall productivity of the city.¹ Match-specific output depends on the Euclidean distance x between the type of an agent's ideal partner and location of his actual partner.

BRW assume $y(x)$ is a linear, decreasing function of x . Here, we assume that *expected* output $\tilde{y}(x)$ is continuous, differentiable, and strictly log concave. This implies that output falls more and more rapidly as we continue to increase x .² To simplify the analysis, we also assume that matches between identical agents are un-productive.³ BRW make a comparable assumption, but they show the results are qualitatively the same when

¹ In our empirical specification, A reflects the utilization of inputs other than labor plus any factors external to the firm.

² This assumption on production technology is sufficient to satisfy the conditions for assortative matching characterized in Shimer and Smith (2000).

³ Let \bar{d} denote the Euclidean distance from the agent's own type and his ideal partner. It is sufficient to assume there exists a value $\bar{x} \leq \bar{d}$ such that $y(\bar{x}) = 0$.

this assumption is violated, or if, instead, it is assumed that output is maximized by matching among identical agents.

Unmatched workers engage in search for a new partner. The arrival rate of meetings between unmatched workers follows a Poisson process. In BRW, the *meeting hazard rate* is simply $\mu = \alpha \cdot U$.⁴ The exogenous parameter α is a measure of the efficiency of the meeting technology, which in turn may depend on the time required to travel to meeting places, the depth of the labor market, or both.

Workers cannot recognize each other's type until after they meet, and this implies that not all meetings result in matches. The probability that a meeting results in a match will depend on the selectivity of workers, which in turn depends on the opportunity cost of declining to form a match in the current meeting. In a symmetric equilibrium, agents will agree to match with any agent whose type is sufficiently close to the ideal, i.e., where $x \leq \delta$, where δ is endogenously determined. Agents choose the value of δ that equates the output from the *marginal* match with the flow value of rejecting the match and waiting for the next encounter: $Ay(\delta) = rV_u$. Let $\beta(\delta, U)$ denote the endogenous *match rate*, the product of the meeting rate μ times the probability that a meeting will result in a match.

Equilibrium

BRW prove the existence of a unique, symmetric, stationary equilibrium of the game for two situations: (1) a city of fixed population and (2) an "open" city where population is endogenously determined. We explore the latter equilibrium, taking into account our assumption about the properties of expected output. Note that uniqueness follows from the fact that agents are distributed uniformly over the type space.⁵

We begin by computing the steady-state value functions holding population constant and assuming that agents choose symmetric strategies. The flow value of being matched to an agent whose type is of distance x from one's own is:

$$r\hat{V}_m(x, U, \delta) = Ay(x) + \pi \left[V_u(U, \delta) - \hat{V}_m \right],$$

which consists of the instantaneous rate of inventions generated by the current match plus the capital loss associated with the cessation of the match, discounted by the arrival rate of the exogenous separation shock.

The agent does not know the exact type of his next partner, so V_u is calculated as an expected value taking into account the agent's selectivity δ . The next match does not

⁴ Note that the meeting technology exhibits increasing returns. BRW argue that the findings of empirical studies are consistent with this assumption, but they also point out that their results do not depend on this assumption.

⁵ The closed city version of BRW can be mapped to a special case of the model in Burdett and Wright (1998), where agents' payoffs from prospective matches are symmetric. Proposition 2 of that paper shows that if the distribution over types is log-concave, there is only one steady-state equilibrium. Proposition 4 shows this holds even where the matching function exhibits increasing returns. Uniqueness is preserved in the open city version of the model because congestion costs are monotonically increasing in population.

arrive immediately after the current one is dissolved, so V_u also reflects the expected amount of time before meeting a suitable partner, and this depends on both U and δ . More explicitly:

$$(A.1) \quad rV_u(U, \delta) = \beta(\delta, U)[V_m(U, \delta) - V_u(U, \delta)],$$

which is simply the discounted present value of the capital gain resulting from the next match. Note that V_m is derived by integrating \hat{V}_m over $x \in [0, \delta]$, holding constant both U and δ . Thus

$$(A.2) \quad rV_m(U, \delta) = A\tilde{y}(\delta) + \pi[V_u(U, \delta) - V_m(U, \delta)],$$

where $\tilde{y}(\delta)$ is the expected value of output from matches when agents' selectivity is δ :

$$\tilde{y}(\delta) = \int_0^\delta y(x)f(x)dx / [1 - F(\delta)] = \frac{1}{\delta} \cdot \int_0^\delta y(x)f(x)dx,$$

where $f(x)$ and $1 - F(\delta)$ are the pdf and cdf, respectively, of the distribution of worker locations. The equality with the right hand side of the expression follows from our assumption that workers are distributed uniformly over a circle of unit circumference.

Substituting for the value functions in (A.1) and (A.2), we find that

$$(A.3) \quad rV_u(U, \delta) = \frac{\beta A \tilde{y}(\delta)}{r + \pi + \beta}.$$

First Order Conditions. Agents that are currently matched enjoy the proceeds of the match until it is dissolved. Unmatched agents are engaged in search and must decide whether to match with the un-matched agents they meet. They choose δ to maximize $rV_u(U, \delta)$. The associated first order condition is

$$(A.4) \quad [r + \pi + \beta]\beta \frac{\partial \tilde{y}}{\partial \delta} + [r + \pi]\tilde{y} \frac{\partial \beta}{\partial \delta} = 0.$$

Holding constant U and the selectivity of other agents, becoming less selective (raising δ) increases the arrival rate of the next partner ($\partial \beta / \partial \delta > 0$) but reduces the expected income generated in the next match ($\partial \tilde{y} / \partial \delta < 0$).

Because agents' types are distributed uniformly over the unit circle and we assume that matches of identical agents are unproductive, $\beta(\delta, U) = 4\delta\mu(U)$. This allows us to rewrite the first order condition:

$$(A.5) \quad \frac{\beta}{r + \pi + \beta} = \frac{y(\delta)}{\tilde{y}(\delta)} \equiv 1 - \eta,$$

where $y(\delta)$ represents the output generated by a marginal match and η is the elasticity of average output with respect to increases in δ .

Lemma 1: There exists an interior solution to the worker's optimization problem characterized by the equality in (A.5).

Proof: Existence follows from the fact that $\tilde{y}(\delta)$, $\tilde{y}(\delta)$ and β are continuous and differentiable. If we take the limit as $\delta \rightarrow 0$, holding U constant, the right-hand side of (A.5) converges to 1 while the left-hand side converges to 0. Conversely, if we take the limit as $\delta \rightarrow \bar{\delta}$, the right-hand side of (A.5) converges to 0, while the left-hand side converges to a value that is strictly greater than zero. Finally, note that the second order condition is simply $[r + \pi + \beta]\beta y'(\delta)/\delta < 0$. ■

The left hand side of (A.5) reflects agents' discount rate and the rate of turnover between the states of being matched and unmatched. The first order condition implies that when the match rate β is relatively large, the ratio of marginal to average output is close to one. The following lemma shows that if $\tilde{y}(x)$ is log-concave, relatively large values of β are associated with relatively small values of δ . This establishes the intuition that, in equilibrium, a higher arrival rate of matches is associated with more selectivity (lower values of δ) even though $\partial\beta/\partial\delta|_U > 0$.

Lemma 2: If $\tilde{y}(x)$ is strictly log-concave, η is strictly increasing in x .

Proof: This follows directly from the definition of $\tilde{y}(\delta)$, which is the expected value of output, conditional on the level of selectivity δ . Taking first and second derivatives of $\text{Ln}(\tilde{y}(\delta))$, we have

$$\text{Ln}(\tilde{y}(\delta))' = \frac{1}{\delta} \left[\frac{y(\delta)}{\tilde{y}(\delta)} - 1 \right] = \frac{-\eta}{\delta} < 0; \quad \text{Ln}(\tilde{y}(\delta))'' = \frac{1}{\delta^2} [\eta - (1-\eta)(\varepsilon - \eta)];$$

where $\varepsilon \equiv -\delta y'(\delta)/y(\delta)$ is the elasticity of *marginal* output with respect to agents' selectivity. Thus strict log concavity of expected output implies that

$$(A.6) \quad \frac{\eta}{1-\eta} < \varepsilon - \eta.$$

A necessary characteristic of the production function, then, is that that marginal output falls proportionately more rapidly than average output as δ rises. In addition, the first order condition implies that $\eta \leq 1$.⁶ But these are sufficient conditions for our proof as

$$\frac{\partial\eta}{\partial\delta} \cdot \delta = \frac{\partial(1 - y(\delta)/\tilde{y}(\delta))}{\partial\delta} = \frac{-y(\delta)}{\tilde{y}(\delta)} \cdot \left\{ \frac{\delta y'(\delta)}{y(\delta)} - \left[\frac{y(\delta)}{\tilde{y}(\delta)} - 1 \right] \right\} = (1-\eta)(\varepsilon - \eta). \quad \blacksquare$$

In steady state the instantaneous measure of agents finding matches is just equal to the measure of agents whose matches are terminated: $\beta(U, \delta)U = \pi M$. Substituting for β and M , we can solve a quadratic equation in U . One root is positive:

⁶ We know there are values of δ where this condition holds because $\lim_{\delta \rightarrow 0} \tilde{y}(\delta) = y(0)$.

$U = \pi(\lambda - 1)/8\alpha\delta$ where $\lambda = \sqrt{1 + 16\alpha\delta N/\pi} \geq 1$. The corresponding match rate is $\beta(U, \delta) = \pi(\lambda - 1)/2$.

Equilibrium Population Mass. To close the model, we formalize the worker's participation constraint:

$$(A.7) \quad rV_u(U, \delta) + C \geq w \cdot N,$$

where agents take U and the selectivity of other agents as given.⁷ C represents the utility derived from the available public goods, net of any lump sum taxes used to finance them. We assume $C \geq 0$. The last term, $w \cdot N$, represents the loss of utility that results from congestion, including the rise in costs that result as scarce local inputs are bid up or depreciate more rapidly.

Taken together, (A.3) and (A.5) imply that $rV_u(U, \delta) = Ay(\delta)$. In other words, the flow value of being an un-matched agent is just equal to the output generated by the marginal match. In equilibrium (A.7) is satisfied with equality, so $N = (Ay(\delta) + C)/w$. This in turn implies that

$$\beta(U, \delta) = \frac{\pi}{2} \left(\sqrt{1 + 16\alpha\delta (Ay(\delta) + C)/\pi w} - 1 \right).$$

Comparative Static Results for Agents' Selectivity and Population

We can substitute the preceding expression for β into our first order condition and derive comparative static results based on a single equation.

Proposition 1: δ is decreasing in α , A , and C , but increasing in w and π .

Corollary: N is increasing in α , A , and C , but decreasing in w and π .

In words, agents become more selective in response to an increase in the efficiency of the matching technology, the average productivity of matches, or local public goods. Agents become less selective in response to an increase in congestion costs or the separation rate.

Proof: For simplicity, we present the comparative static results in terms of elasticities. We begin with the result for α , the arrival rate of meetings per unmatched worker:

$$\frac{\partial \delta}{\partial \alpha} \cdot \frac{\alpha}{\delta} = -\eta \left(\frac{\lambda + 1}{2\lambda} \right) / \left\{ \eta \left(\frac{\lambda + 1}{2\lambda} \right) (1 - \phi) + (\varepsilon - \eta) \right\} < 0,$$

where $\phi = -Ay'(\delta)\delta/(Ay(\delta) + C)$ is the elasticity of population with respect to changes in δ . We do not wish to rule out the possibility that $\phi > 1$, so we must verify that the denominator in the preceding expression is indeed positive. We begin by noting that $\varepsilon \geq \phi$ because we have assumed that $C \geq 0$. It is then sufficient to verify that

⁷ The net benefit of living outside the city is normalized to zero.

$$\left(\frac{\lambda+1}{2\lambda}\right)\eta(1-\varepsilon)+(\varepsilon-\eta)>0.$$

Dividing by η and rearranging terms we have

$$\frac{\varepsilon}{\eta}\left(1-\left(\frac{\lambda+1}{2\lambda}\right)\eta\right)-\left(1-\left(\frac{\lambda+1}{2\lambda}\right)\right)>0,$$

which is positive if $\tilde{y}(\delta)$ is strictly log concave.

Next, we consider the effect of increasing w , the coefficient on population that reflects utility lost to congestion:

$$\frac{\partial\delta}{\partial w}\cdot\frac{w}{\delta}=\frac{\partial\delta}{\partial\pi}\cdot\frac{\pi}{\delta}=\eta\left(\frac{\lambda+1}{2\lambda}\right)\left/\left\{\eta\left(\frac{\lambda+1}{2\lambda}\right)(1-\phi)+(\varepsilon-\eta)\right\}>0.$$

This is also the expression for the elasticity of δ with respect to changes in the separation rate π . Now we consider an increase in A , the overall productivity of matches:

$$\frac{\partial\delta}{\partial A}\cdot\frac{A}{\delta}=-\eta\left(\frac{\lambda+1}{2\lambda}\right)\cdot\left(\frac{Ay(\delta)}{Ay(\delta)+C}\right)\left/\left\{\eta\left(\frac{\lambda+1}{2\lambda}\right)(1-\phi)+(\varepsilon-\eta)\right\}<0,$$

where the extra term in the numerator is the elasticity of population with respect to A .

Finally, we consider the effect of an increase in public goods available in the city:

$$\frac{\partial\delta}{\partial C}\cdot\frac{C}{\delta}=-\eta\left(\frac{\lambda+1}{2\lambda}\right)\cdot\left(\frac{C}{Ay(\delta)+C}\right)\left/\left\{\eta\left(\frac{\lambda+1}{2\lambda}\right)(1-\phi)+(\varepsilon-\eta)\right\}<0,$$

where the extra term in the numerator is the elasticity of population with respect to C .

The participation constraint (A.7) can be rewritten as $Ay(\delta)+C=wN$. Thus equilibrium population is inversely related to δ , which proves the corollary. ■

Comparative Static Results for Per Capita Output and Population Mass

Here we present the main results characterized in our companion working paper:

Proposition 2: Per capita output of inventions is increasing in α , A , and C , but decreasing in w and π .

Corollary: Per capita output of inventions is higher in cities with a larger population mass.

Proof: Per capita output is the product of the share of the population that is matched and the average productivity of matches:

$$\Phi\equiv\frac{M}{N}\cdot A\tilde{y}(\delta)=\frac{\beta}{\pi+\beta}\cdot A\tilde{y}(\delta)=\left(\frac{\lambda-1}{\lambda+1}\right)\cdot A\tilde{y}(\delta).$$

Again we present comparative static results in terms of elasticities, which take the form:

$$\frac{d\Phi}{dz} \cdot \frac{z}{\Phi\lambda} = \frac{z}{\lambda(\lambda^2-1)} \cdot \frac{\partial\lambda^2}{\partial z} + (1-\phi) \cdot \frac{\partial\delta}{\partial z} \cdot \frac{z}{\delta} - \eta\lambda \cdot \frac{\partial\delta}{\partial z} \cdot \frac{z}{\delta}.$$

The first two terms reflect the change in the share of agents matched at any given time (M/N). These reflect changes in the intensive margin (holding N constant, does M rise or fall?) and the extensive margin (holding constant (M/N), does N rise or fall?). The last term reflects the change in expected output per match, characterized in the previous proposition. Rearranging the expression, we have

$$\frac{d\Phi}{dz} \cdot \frac{z}{\Phi\lambda} = \frac{z}{\lambda(\lambda^2-1)} \cdot \frac{\partial\lambda^2}{\partial z} + \{(1-\phi) - \eta\lambda\} \cdot \frac{\partial\delta}{\partial z} \cdot \frac{z}{\delta}.$$

Consider first an increase in the arrival rate of meetings per unmatched worker α :

$$\frac{d\Phi}{d\alpha} \cdot \frac{\alpha}{\Phi\lambda} = 1 + \{(1-\phi) - \eta\lambda\} \cdot \frac{\partial\delta}{\partial\alpha} \cdot \frac{\alpha}{\delta} = \frac{\left(\frac{\lambda+1}{2}\right)\eta^2 + (\varepsilon - \eta)}{\left(\frac{\lambda+1}{2\lambda}\right)\eta(1-\phi) + (\varepsilon - \eta)} > 0.$$

Next consider an increase in the overall productivity of innovations A :

$$\frac{d\Phi}{dA} \cdot \frac{A}{\Phi\lambda} = \frac{Ay(\delta)}{Ay(\delta) + C} + \{(1-\phi) - \eta\lambda\} \cdot \frac{\partial\delta}{\partial A} \cdot \frac{A}{\delta} = \frac{\left(\frac{Ay(\delta)}{Ay(\delta) + C}\right) \left\{ \left(\frac{\lambda+1}{2}\right)\eta^2 + (\varepsilon - \eta) \right\}}{\left(\frac{\lambda+1}{2\lambda}\right)\eta(1-\phi) + (\varepsilon - \eta)} > 0.$$

Consider the effect of an increase in public goods C available in the city:

$$\frac{d\Phi}{dC} \cdot \frac{C}{\Phi\lambda} = \frac{C}{Ay(\delta) + C} + \{(1-\phi) - \eta\lambda\} \cdot \frac{\partial\delta}{\partial C} \cdot \frac{C}{\delta} = \frac{\left(\frac{C}{Ay(\delta) + C}\right) \left\{ \left(\frac{\lambda+1}{2}\right)\eta^2 + (\varepsilon - \eta) \right\}}{\left(\frac{\lambda+1}{2\lambda}\right)\eta(1-\phi) + (\varepsilon - \eta)} > 0.$$

Finally, consider the effect of an increase in the utility lost due to congestion as population increases w (The effect of an increase in the separation rate π takes on exactly the same expression):

$$\frac{d\Phi}{dw} \cdot \frac{w}{\Phi\lambda} = -1 + \{(1-\phi) - \eta\lambda\} \cdot \frac{\partial\delta}{\partial w} \cdot \frac{w}{\delta} = \frac{-\left\{ \left(\frac{\lambda+1}{2}\right)\eta^2 + (\varepsilon - \eta) \right\}}{\left(\frac{\lambda+1}{2\lambda}\right)\eta(1-\phi) + (\varepsilon - \eta)} < 0.$$

The corollary then follows from the previous corollary. ■

Comparative Static Results for Changes in Land Area

In BRW population varies, while the physical size of the city is fixed. In other words, in terms of the model, changes in population and density are the same. Thus, in our empirical work it is important for us to examine the effects of both city size (the number of workers) and density (the number of workers per square mile).

But we also wish to consider the possibility that exogenous increases in land area might reduce density and therefore per capita output. This will occur if an insufficient number of workers respond to the reduction in congestion by moving into the city. To explore this possibility, we assume that the efficiency of the meeting rate parameter α and congestion coefficient w are decreasing in city land area L . We show the following:

Proposition 3: Increasing L decreases (increases) δ , and Φ if α is more (less) sensitive to changes in L than is w .

Proof: We derive the elasticity of per capita output with respect to changes in land area in the same manner as we did in Propositions 2 and 3:

$$\frac{d\Phi}{dL} \cdot \frac{L}{\Phi} = \frac{L}{\lambda} \left(\frac{\alpha'(L)}{\alpha(L)} - \frac{w'(L)}{w(L)} \right) \left\{ 2 + \frac{(\lambda+1)(1-\eta)\eta^2}{\left(\frac{\lambda+1}{\lambda}\right)\eta(1-\phi) + 2(\varepsilon-\eta)} \right\}. \blacksquare$$

In the model, increasing land area has two effects on workers' incentive to migrate to the city. The first effect is a reduction in the costs associated with congestion which, all else equal, induces additional migration. The second is a reduction in expected income that results from waiting longer for the next meeting with an unmatched worker. All else equal, this would discourage workers from entering the city.

It turns out that, in the absence of a fully specified model for the supply of developed land, we cannot sign the effect a priori. If the reduction in congestion costs is sufficiently large relative to the decline in the efficiency of meeting technology, both density and per capita output rises. But if the reduction in congestion costs is relatively small, density and per capita output will fall.

APPENDIX B: Data Construction

Patent Data

Our patent data are derived from data sets furnished by the Technology Forecasting and Assessment Branch of the U.S. Patent and Trademark Office. These include the US Patent Inventor File 1977-99: All Grants; 75-76 Utility Patents Only and the PATSIC99 file. We are grateful to Jim Hirabayashi of USPTO for his assistance with these data.

We assemble the home address information of the first inventor named on each patent from the inventor's file. We began with 1,198,376 patents granted to inventors with an American address over the years 1975-1999. We assigned a state-county FIPS code for each patent by matching address information against the 1998 vintage of NIST's FIPS55 place names data set. We obtained unique ZIP code-county and place name-county

matches for 81 percent of the patents. For another 8.5 percent of the patents, we could not initially identify a unique county for the inventor but we could identify a unique metropolitan area. Another 6.6 percent of patents were geographically located via matches against the location of an R&D lab of the firm owning the patent and manual searches.

In all 1,155,133 patents were matched to a county or an MA, while 43,243 were not. Of those patents that were successfully placed in an MSA, 581,001 were granted between 1990 and 1999 (inclusive); 46,647 were located outside an MSA. That leaves 534,354 patents in MAs that were used to construct the patent variable.

Patent classifications are obtained by matching patents in the inventor file to patents in the firm file. The patent class dummies (medical, chemical, computer, electrical, and mechanical) are constructed using the primary patent classes suggested in Appendix 1 of Hall, Jaffe, and Trajtenberg (2001). The industry mix of patents is obtained by matching patent numbers with those included in the NBER Patent Citations Data File, obtaining the CUSIP of the firm initially assigned the patent and obtaining the SIC of that firm from the 1999 vintage of Compustat. The high-tech patent share is derived using the set of R&D intensive three-digit SIC industries identified in Office of Technology Policy (2001).

Our Definition of Metropolitan Areas

Our MA definitions are based primarily on MSA definitions defined by the Office of Management and Budget in 1983 (<http://www.census.gov/population/estimates/metro-city/83mfips.txt>). Several adjustments are made:

- The six MSAs in Puerto Rico are removed.
- New England County Metropolitan Areas (NECMAs) are used as our MAs in New England.
- The Bureau of Economic Analysis (BEA) uses its own set of county-equivalent codes to tabulate data for independent cities and their surrounding counties together. For all data from the Regional Economic Information System (REIS), data for our MAs are built up including these independent cities.
- Nine Consolidated Metropolitan Statistical Areas (CMSAs) are employed instead of their 25 component MSAs: Chicago, IL-IN-WI; Cincinnati-Hamilton, OH-KY-IN; Cleveland-Akron-Lorain, OH; Dallas-Fort Worth, TX; Houston-Galveston-Brazoria, TX; Kansas City, KS-MO; Portland-Vancouver, OR-WA; Seattle-Tacoma, WA; and St. Louis- East St. Louis- Alton, IL-MO.
- Seven ad hoc metropolitan areas were also created: Denver-Boulder-Greeley (Boulder-Longmont, CO PMSA; Denver, CO PMSA; & Greeley, CO MSA); Greenville-Anderson (Anderson, SC MSA and Greenville-Spartanburg, SC MSA); Los Angeles-Anaheim (Anaheim-Santa Ana, CA PMSA and Los Angeles-Long Beach, CA PMSA); Midland-Odessa (Midland, TX MSA and Odessa, TX MSA); New York-Northern New Jersey (Bergen-Passaic, Jersey City, Middlesex-Somerset-Hunterdon, Monmouth-Ocean, Nassau-Suffolk, New York, Newark, Orange County); San Francisco-Oakland (Oakland, CA PMSA and San Francisco, CA PMSA); and Sarasota-Bradenton (Bradenton, FL MSA and Sarasota, FL MSA).

We used these definitions for certain cities rather than the underlying MSAs because two or more cities shared a common border and we could not always assign some patents (a few thousand) to an MSA with certainty (see Patent Data above). If we used the underlying MSAs, we would either have to discard these patents or take the chance that the error rate in assigning patents to cities might vary across cities. The methodology used in USPTO (2000) assigns equal shares of patents to counties with common place names when a patent cannot be matched to a unique county. This too might imply a higher error rate in assigning patents when MSAs are close to each other.

We compared our MSA patent counts to those reported in USPTO (2000) and found them to be extremely close except for a few instances. In some cases two or more MSAs were in close proximity (e.g., Dallas and Fort Worth). In a few others, the place name of the inventor's address was common to more than one county, regardless of distance. The PTO algorithm divided those patents equally across those counties.

Our definition of metropolitan areas or the manner of allocating patents to MSAs does not significantly influence our results. In an earlier version of this paper (Carlino, Chatterjee, and Hunt 2001) we estimated the relationship between patent intensity and employment density using a data set of 296 MSAs and PMSAs as defined by OMB in 1983 and using patent counts built up from the data contained in USPTO (2000). The results were qualitatively the same as those reported here, although the estimated coefficients were somewhat larger.

Missing Data

Four MSAs are dropped in the analysis because of missing data. One MSA (Enid, OK) does not have a corresponding urbanized area. Owing to disclosure limitations, a Herfindahl index of industry employment shares cannot be calculated for Atlantic City, NJ, and Tallahassee, FL. In addition, manufacturing employment is not available in 1989 for Columbus, GA-AL.

Geographic Variables

Urbanized Area land area for every county was obtained from Table 34 of Census Bureau (1993a). We also obtained comparable measures for urbanized areas defined in 1980 and 1970 from Census Bureau (1979, 1984).

There are a few instances where a county includes land area associated with more than one urbanized area. For example, portions of Bucks County, PA, are associated with the Philadelphia and Trenton urbanized areas. To be consistent with our other county-based measures, we attribute this land area (and associated employment) to the MA associated with that county (e.g., Philadelphia).

The weather and topography instrumental variables are based on the USDA's Natural Amenity Scale project (<http://www.ers.usda.gov/data/naturalamenities>). The data are reported at county level, which we aggregate, based on county land area, to MSAs. We use the variables indicating, for the years 1941-1970, mean hours of sunlight and temperature in January, mean temperature in July, the percent of land area covered by surface water, and five dummy variables for the presence of a particular type of geography (plains, tablelands, open hills and mountains, hills and mountains, and plains with hills and mountains) built up from a finer gradation in the ERS data. We also

included as an instrument the amount of MA area covered by water in 1990. Those data were obtained from a county level tabulation reported in the Census Bureau's Gazetteer (<http://www.census.gov/geo/www/gazetteer/gazette.html>).

Economic and Demographic Variables

Our primary employment data are county-level values reported in the BEA's REIS database and aggregated to the MA level. Those data are derived from the BLS Covered Employment and Wages Program (ES-202), which represents the average annual number of full- and part-time jobs held by all workers who are covered by unemployment insurance. Industry breakdowns are based on 1987 SIC definitions.

Our measures of residency-based employment in 1990 were obtained from the Census Bureau web site (www.americanfactfinder.com). Those data are derived from the STF3 (5 percent sample) tape. A separate count was obtained for every county that includes an urbanized area. These are aggregated to MAs in the same manner as our other variables.

Our counts of employment for professional specialty occupations are derived from the 1990 census (STF3). This is a residency-based count, but here we include all residents in the counties making up an MA, not just residents in the urbanized area portion of those counties. The occupation codes for this category of jobs are codes 043-202 in the 1990 Census Occupation Classification System.

Our counts of scientists and engineers are residency-based measures of workers in these occupations living in urbanized areas as reported in Table 34 of Census Bureau (1993c). We aggregated these counts to be consistent with our MA definitions.

The shares of MSA land area, population, and employment contained in urbanized areas discussed on pages 13-14 are derived from Tables 48, 50-51 of Census Bureau (1993a) and Table 33 of Census Bureau (1993b) and Census Bureau (1993c).

The Herfindahl-Hirschman Index (HHI) is calculated for 1989 and includes employment for ten industrial sectors. These include Construction, Manufacturing, Transportation and Public Utilities, Wholesale Trade, Retail Trade, Finance, Insurance, and Real Estate, Services, Civilian Federal Government, State and Local Government, and a category, other, that consists primarily of employment in the military, agriculture, and mining. The HHI is calculated at the MA level. Wherever county data are missing for 1989, either data from the previous or following year are used or MSA-level data are substituted when appropriate.

The percentage of population with a college degree or more education is derived from the 1990 Census American Fact Finder. Because these are MSA-level data, it was necessary to create a weighted average of component MSAs for the special CMSAs we created; this was done using 1990 Census Bureau mid-year population estimates aggregated from county level to MSA. The population data were also used to calculate our measure of patents per capita.

Our instrumental variables include a measure of the number of restaurants and museums and crimes in 1989. These are derived from county-level data, as reported in County Business Patterns, and aggregated to the MA level. They are converted to intensities by dividing by population in 1989.

Research and Development Variables

The amount of academic R&D in each MSA is derived from the National Science Foundation's (NSF) "Academic Science and Engineering: R&D Expenditures," as archived in the WebCaspar search engine at the NSF web site. Expenditures are averaged over the period 1987-89 and are normalized by the total fall enrollment as reported in the Integrated Postsecondary Education Data System assembled by the National Center for Education Statistics and archived on WebCaspar. S&E expenditures reported for a number of university systems were also allocated to particular campuses using advanced S&E degrees granted by those campuses (details are found in an extended data appendix available from the authors). Expenditures are built up from counties according to our MA definitions.

Data on the location and resources of the Federally Funded Research and Development Centers were provided to us by Ronald Meeks of the National Science Foundation.

Data on private R&D facilities were extracted from the 1989 edition of the *Bowker Directory of American Research and Technology*. They are matched to the SIC of the parent company using the 1999 vintage of Compustat.

Trade Secrecy Index

We assigned the industry-specific effectiveness rating in Table 1 of Cohen, Nelson, and Walsh (2000) to two-digit or three-digit SIC industries. These ratings are a categorical response to a question that asks for the proportion of product innovations for which trade secrets are effective in preserving the resulting profits. For each MA, we compute a weighted average of the industry ratings using as weights the shares of all private R&D facilities in the MA contained in those industries.

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